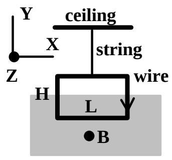
Physics 2402 Test #3 (8:30 Class) Fall 1998

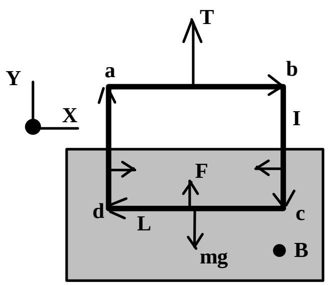


1 A rectangular loop of wire having the dimensions L = 0.500 m and H = 0.300 m has a mass of 0.250 kg. The loop, which is at rest, is connected by a string to the ceiling. The X and Y axes are shown in the figure and the +Z direction is out of the page. The shaded region has a uniform magnetic field of 0.200 Tesla magnitude. The direction of the field is out of the page in the +Z direction and one half of the loop's area is inside the shaded field region.

(a) If the loop is carrying a current of 7.50 A in the clockwise direction as shown in the figure, determine the total magnetic force (be sure to specify its direction with the proper unit vectors) exerted on the loop and find the tension in the string.

(b) If the loop is now moved down so that it is completely inside the shaded field region find the tension in the string.

Solution:



1 (a) The force on side "ab" is zero since the field is zero. The forces on the sides "bc" and "da" are in the directions shown. Since their magnitudes are the same the net force on the sides is zero. For the side "cd" we have: **L** = .5(-**i**)m and **B** = .2**k** (T). The magnetic force on "cd" is:

F = I(**L** x **B**) and **F** = -7.5(.5)(.2)(**i** x **k**).

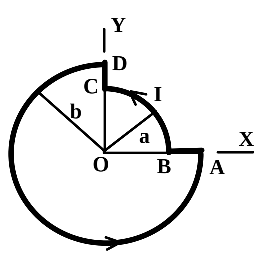
Since **i** x **k** is -**j** we have **F** = 0.75**j** (N).

This force is shown in the figure. This can also be obtained by finding the magnitude of the magnetic force (ILBsin90) and applying the right hand rule to obtain the direction. Since the loop is at rest we obtain from the Y component of Newton's II Law:

T + F - mg = 0 and solving gives T = 1.70 N.

(b) When the loop is completely inside the field region the magnetic force on "ab" is easily found to be: **F** = 0.75**j** (N). and the net magnetic force on the whole loop is zero. By Newton's II Law:

T - mg = 0 and T = 2.45 N.



2 A wire is bent into the shape "ABCDA" as shown. "BC" is part of a circle of radius "a" while "DA" is part of a circle of radius "b". The center of both of these circles is at "O" and the angle "AOD" is 90 degrees or $\pi/2$ radians. A current is flowing in the wire in the direction A to B to C to D and back to A.

(a) **Derive**, using the Biot-Savart Law, the magnitude of the magnetic field produced by the wire segment "BC" at the origin. Be sure to give the correct direction using the proper unit vectors. The +Z direction is out of the page.

(b) What is the magnetic field at the origin produced by the current in the straight section "AB"?

(c) Find the total field at the origin produced by the complete loop "ABCDA". Be sure to give its direction using the proper unit vectors.

Solution:

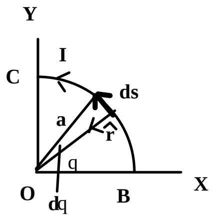


Fig. a

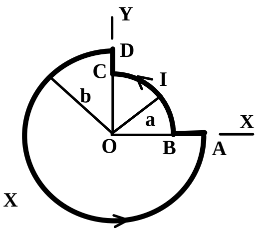


Fig. b

2 (a) In Fig. a the vectors required to calculate the field at the origin are shown. The magnetic field, d**B** , caused by the current in the small element **ds** is found using the Biot-Savart Law:

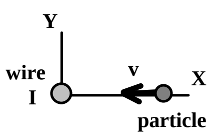
$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi a^2} = \frac{\mu_0 I ds(1)\sin 90}{4\pi a^2}\hat{k} = \frac{\mu_0 I a d\theta}{4\pi a^2}\hat{k}$$

where we have used ds = adθ and the right hand rule to find that the direction of the vector cross product is out of the page in the positive Z direction. The total field at the origin is obtained by integrating d**B**:

$$\vec{B} = \hat{k}\frac{\mu_0 I a}{4\pi a^2}\int_0^{\pi/2} d\theta = \hat{k}\frac{\mu_0 I}{4\pi a}\left(\pi/2\right) = \hat{k}\frac{\mu_0 I}{8a}$$

(b) On the straight segments $d\vec{s} \times \hat{r}$ is zero because the angle between the two vectors is 0 or 180 degrees and the fields are zero.

(c) The field due to the loop CA is found from the result of part (a) except the radius "B" is used and the angle is 3π/2. The result is $\hat{k}\frac{3\mu_0 I}{8b}$. The total field is: $\vec{B} = \hat{k}\frac{\mu_0 I}{8}\left(\frac{1}{a} + \frac{3}{b}\right)$



3 The X and Y axes are shown in the figure while the positive Z axis is coming out of the page. A very long straight wire is placed along the Z axis (from minus infinity to plus infinity) and is carrying a constant current whose magnitude and direction are not known. At a particular time a small particle having a positive charge of $3.00\text{x}10^{-3}$ C is on the positive X axis moving directly towards the wire with a speed of 200 m/s . At this same time it is a distance of 0.05 meters from the wire (at the location shown in the figure) and it experiences a magnetic force of $1.20\text{x}10^{-4}$ N directed out of the page, in the positive Z direction. The magnetic field that causes this force is created by the current in the long wire.

(a) Find the magnitude and the direction of the magnetic field at the location of the charge. Express the magnetic field using the proper unit vectors.

(b) Find the magnitude of the current in the wire and specify its direction as being into or out of the page.

Solution:

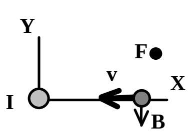


Fig. a

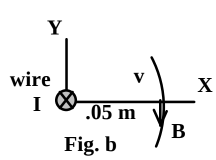


Fig. b

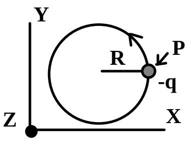
(a) Since the charge is positive, the magnetic force is in the same direction as the vector cross product of **the velocity and the field.** In Fig. a the magnetic field must be in the negative Y direction so that the vector cross product $\vec{v} \times \vec{B}$ and the magnetic force will be in the positive Z direction. The magnitude of the magnetic force is:

$F = |qvB\sin 90| \quad \Rightarrow \quad B = \frac{F}{|qv|} = 2.00 \times 10^{-4}\,T$. The magnetic field is: $\vec{B} = -2.00 \times 10^{-4}\,\hat{j}(T)$.

(b) From part (a) we know that the field produced by the wire must be in the direction shown in Fig. b. The current must be into the page to generate a field in that direction. From Ampere's Law the magnitude of the field produced by a long wire carrying a current "I" a distance "r" from it is:

$B\int ds = m_0 I \Rightarrow B = \frac{m_0 I}{2p\, r} \Rightarrow I = \frac{2p\, rB}{m_0} = 50.0A$. The current is 50.0 A into the page.

Physics 2402 Test #3 (1:30 Class) Fall 1998



1 A uniform magnetic field, which is perpendicular to the XY plane, is causing a negative charge (- 3.00x10^{-4} C) to move in a circular path of radius R (0.200 m) in the XY plane with a constant speed of 3.50x10^{2} m/s. The mass of the charge is 6.00x10^{-8} kg and the positive Z axis is out of the page.

(a) Find the magnetic field and be sure to specify its direction using the appropriate unit vectors.

(b) At the instant the charge is at point P (moving in the +Y direction) a uniform electric field is switched on (the magnetic field is not changed) and the charge then moves along a straight line parallel to the Y axis with the same constant speed 3.50x10^{2} m/s. Find this electric field and be sure to specify its direction using the appropriate unit vectors.

Solution:

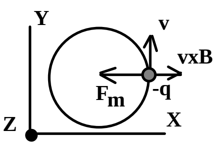


Fig. a

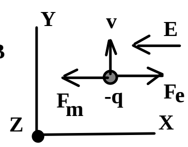


Fig. b

1 (a) Fig. (a) shows the velocity and magnetic force vectors at a particular point on the circular path. Since the charge is negative, $\mathbf{F}_m$ must be in the direction opposite to the vector cross product **v** x **B** as shown in the figure. For **v** x **B** to be in the the direction shown **B** must be out of the page in the positive Z direction. The magnitude of the magnetic force is:

$F_m = |q|vB\sin 90 = |q|vB$.

Applying Newton's II Law to the circular motion of the charge gives:

$$F_m = \frac{mv^2}{R} \quad \Rightarrow \quad |q|vB = \frac{mv^2}{R} \quad \Rightarrow \quad B = \frac{mv}{|q|R}$$

Solving for the field we find that B = 0.35 T and as a vector: $\vec{B} = .35\hat{k}(T)$

(b) In Fig. (b) An electric field has been applied to make the charge move with a constant velocity in the direction shown. The electric force, $\mathbf{F}_e$, must be equal in magnitude but in the opposite direction to $\mathbf{F}_m$. For example if the electric force had a component in the Y direction the charge would experience an acceleration but we know that the velocity remains constant. The magnitude of the electric force is:

$$F_e = F_m \quad \Rightarrow \quad |q|E = |q|vB\sin 90 \quad \Rightarrow \quad E = vB$$

Solving we find that E = 1.23x10^{2} (N/C) and since the charge is negative the direction of the field is opposite to the force and $\vec{E} = -1.23\times10^{2}\,\hat{i}\,(N/C)$

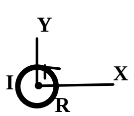


Fig. a

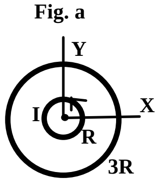


Fig. b

2 (a) A circular wire loop of radius "R" is carrying a current "I" in the counter clockwise direction as shown in Fig. "a". Its center is at the origin. The X and Y axes are in the plane of the page while the +Z axis is out of the page. **Derive** , using the Biot-Savart Law, the magnetic field produced by the loop at the origin. Be sure to express this field as a vector using the proper unit vectors.

(b) The loop mentioned in part (a) is now surrounded by a concentric circular wire having a radius "3R" as shown in Fig. "b". The total field of both loops measured at the origin is now zero. Find the current (expressed in terms of the given current I) in the loop having the radius "3R" and give (with an explanation) its direction (clockwise or counter

clockwise).
Solution:

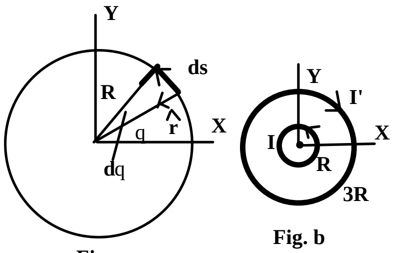


Fig. a Fig. b

2 (a) Fig. a shows only the inner wire loop with the appropriate vectors indicated. The magnetic field at the center due to the small element **ds** is:

$$d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi R^2} = \hat{k}\frac{\mu_0 I ds \sin 90}{4\pi R^2}$$

The arc length ds is also Rdθ so:

$$B = \frac{\hat{k}\mu_0 IR}{4\pi R^2}\int_0^{2\pi} d\theta = \frac{\hat{k}\mu_0 I}{2R}$$

(b) As shown in Fig. (b) the current I' in the outer loop must be in the direction opposite to the current I in the inner loop to produce a field in the -Z direction which is necessary if the total field is zero. The field of the outer loop must have the same magnitude as the inner loop. The magnitude of the field produced by the outer loop at its center is μ_0 I'/(6R). Since the magnitudes of the two fields must be the same:

$$\frac{\mu_0 I'}{6R} = \frac{\mu_0 I}{2R} \quad \Rightarrow I' = 3I$$ and the direction is clockwise.

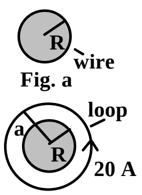


Fig. a

Fig. b

3 (a) In Fig. (a) a very long straight wire of radius R is oriented perpendicular to the page and is carrying a current I out of the page. This current is uniform over the cross sectional area, πR^2, of the wire. Using Ampere's Law **derive** the magnitude of the magnetic field **inside** the wire at a distance "r" from its center (0 < r < R). Compute the field's magnitude if: I = 500 A, R = .020 m and r = .010 m.

(b) In Fig. (b) the same wire is surrounded by a single, concentric, loop of wire which has a radius "a" of .030 m. The loop has a current of 20.0 A in the direction shown. Derive the magnitude of the force "dF" exerted, by the long wire, on a small length element "ds" of the loop then determine the total force exerted on the complete loop by the long wire.

Solution:

3 (a) In Fig. (a) we choose the path "C" to be the circle of radius "r" which is inside the wire. The direction of the magnetic field is tangent to this path as shown. The magnitude of the current density is: J = I/(πR^2) and the current inside this circle is: I = J(πr^2) = Ir^2/R^2. Applying Ampere's Law gives:

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0\left(\frac{Ir^2}{R^2}\right) \Rightarrow B(2\pi r) = \mu_0\left(\frac{Ir^2}{R^2}\right)$$

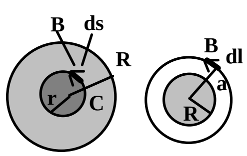


Fig. a Fig. b

and solving for the field we have: $B = \frac{\mu_0 Ir}{2\pi R^2} = 2.50\times 10^{-3}T$

where the numerical value was obtained using r = .01m, R = .02 m and I = 500A.

(b) In Fig. (b) the field created by the current in the straight wire is parallel to the direction of the length **dl** on the circular loop. The magnitude of the force, dF, on this element is:

$$d\vec{F} = Id\vec{l}\times\vec{B} \Rightarrow dF = I(dl)B\sin 0 = 0$$

The total force on the complete loop is therefore zero.