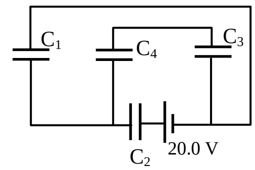
Physics 2402 Test 2 (8:30 Class) Fall 2007

1 (a) The capacitors C_1 ($1.00\text{x}10^{-6}$ F), C_2 ($2.00\text{x}10^{-6}$ F), C_3 ($3.00\text{x}10^{-6}$ F) and C_4 ($4.00\text{x}10^{-6}$ F) are connected to a 20.0 (V) battery as shown in the diagram.
(a) Find the equivalent capacitance.
(b) Find the potential difference across C_2.
(c) Find the charge on C_4.



Solution:
(a) The capacitors C_4 and C_3 are in series and are replaced by C' in Fig. a:

$$\frac{1}{C'}=\frac{1}{C_3}+\frac{1}{C_4} \quad \Rightarrow \quad C'=1.71\times 10^{-6}(F)$$

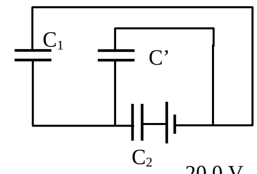


Fig. a

The capacitors C' and C_1 are in parallel and are replaced by C" in Fig. b:
C" = C' + C_1 = $2.71\text{x}10^{-6}$ (F).
Finally the capacitors C" and C_2 in Fig. b are in series and can be replaced by C"' shown in fig. c:

$$\frac{1}{C'''}=\frac{1}{C''}+\frac{1}{C_2} \quad \Rightarrow \quad C'''=1.15\times 10^{-6}(F)$$

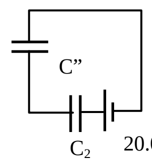


Fig.b

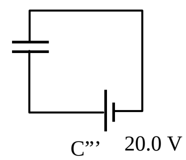


Fig.c

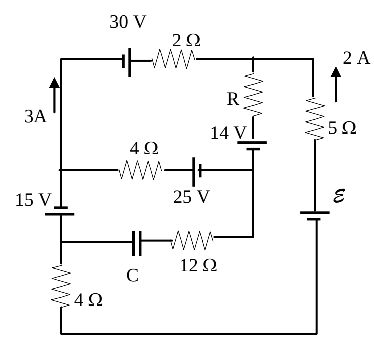
(b) The charge on C"' is: Q'" = C'"ΔV'" = $1.15\text{x}10^{-6}$ (20) = $2.30\text{x}10^{-5}$ (C). Since C"' replaced C" and C_2 in series each of these capacitors have the same charge $2.30\text{x}10^{-5}$ (C). The potential difference across C_2 is:

$$\Delta V_2=\frac{Q_2}{C_2}=11.5(V)$$

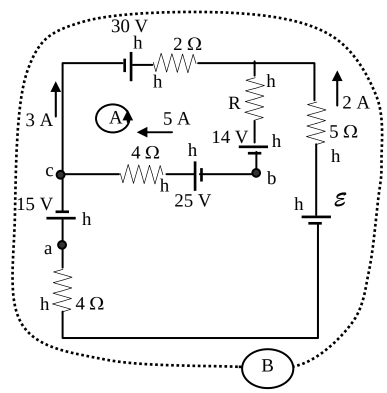
(c) The potential difference across C" is:

$$\Delta V''=\frac{Q''}{C''}=\frac{2.3\times 10^{-5}}{2.71\times 10^{-6}}=8.50(V)$$

Since C" replaced C' and C_1 in parallel, each of these capacitors has the same potential difference as C". The charge on C' is:
Q' = C'ΔV' = $1.71\text{x}10^{-6}$(8.5) = $1.45\text{x}10^{-5}$ (C).
Since C' replaced C_4 and C_3 in series each of these capacitors has the same charge as Q'. The charge on C_4 is $1.45\text{x}10^{-5}$ (C).



2 In the circuit diagram the values of several resistances and battery *Emfs* are shown. Two currents along with their directions are also shown and the capacitor is fully charged. Assume all the given values have three significant figures.
(a) Find the unknown resistance "R".
(b) Find the unknown *Emf* "ℰ".
(c) If the energy stored in the capacitor is $5.00\text{x}10^{-4}$ (J), find its capacitance.



Solution:
(a) The capacitor and the 12 ohm resistor that are connected between "a" and "b" are not shown since no current exists in this part of the circuit because the capacitor is fully charged. We calculate the current in the 4 ohm resistor by applying Kirchoff's first rule at the junction

“c”. This current must be in the direction shown since the other current are moving away from that junction.

$$\sum I_{in} = \sum I_{out} \quad \Rightarrow \quad I = 2 + 3 = 5(A)$$

We apply Kirchoff’s second rule to the loop “A” going counterclockwise. The high potential sides of each resistor and battery are shown in the diagram.

$$\sum_{loop\,A} \Delta V = 0 \quad \Rightarrow \quad +4(5) - 25 + 14 + 5R + 3(2) - 30 = 0 \quad \Rightarrow \quad R = 3.00(\Omega)$$

(b) We apply Kirchoff’s second rule to the loop “B” going counterclockwise.

$$\sum_{Loop\,B} \Delta V = 0 \quad \Rightarrow \quad +15 - 2(4) + Emf - 2(5) + 3(2) - 30 = 0 \quad \Rightarrow \quad Emf = 27.0(V)$$

(c) The potential difference $V_a - V_b$ is found by starting at “b” and moving through any part of the circuit to reach “a”. We choose the direct path:

$$V_a - V_b = \sum_{b \to a} \Delta V = +25 - 5(4) + 15 = 20.0(V)$$

Since the potential difference across the 12 ohm resistor in series with the capacitor is zero (no current in it) the potential difference across the capacitor is 20.0 (V). We can find the capacitance since we know the energy stored:

$$U = \frac{1}{2} Q\Delta V = \frac{1}{2}(C\Delta V)\Delta V = \frac{1}{2} C(\Delta V)^2 \quad \Rightarrow \quad C = \frac{2U}{(\Delta V)^2} = 2.50 \times 10^{-6}(F)$$

3 (a) An egg timer circuit is shown in Fig. a. The capacitor “C” (6.00×10^{-6} F) is initially uncharged with the switch “S” open. Another part of the circuit, which is not shown in Fig. a, measures the potential difference across the capacitor and sounds a buzzer when this potential difference reaches 4.00 (V). If the buzzer sounds three minutes after “S” is closed, find the resistance “R”.
(b) The power being dissipated as heat in the four ohm resistor shown in Fig. b is 1.60×10^{-1} (Watts). Find: (i) the equivalent resistance of this circuit and (ii) the $\mathcal{E}$mf of the battery.

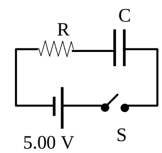


Fig. a

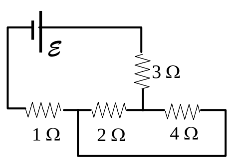


Fig. b

Solution:
(a) When the capacitor is completely charged there is no current in the circuit and the potential difference across the capacitor if the battery potential ΔV_b and the charge is: $Q = C\Delta V_b$. The potential difference across the capacitor at a time “t” after “S” is closed is:

$$\Delta V_c(t) = \frac{q(t)}{C} = \frac{Q}{C}\left(1 - e^{\frac{-t}{\tau}}\right) = \frac{C\Delta V_b}{C}\left(1 - e^{\frac{-t}{\tau}}\right) = \Delta V_b\left(1 - e^{\frac{-t}{\tau}}\right)$$

At a time of 180 (s) the potential difference is 4.00 (V). We can find “τ” and “R”:

$$\frac{\Delta V_c(t)}{\Delta V_b} = \left(1 - e^{\frac{-t}{\tau}}\right) \quad \Rightarrow \quad e^{\frac{-t}{\tau}} = 1 - \frac{\Delta V_c(t)}{\Delta V_b} \quad \Rightarrow \quad e^{\frac{-180}{\tau}} = 1 - \frac{.4}{.5} \quad \Rightarrow \quad \frac{-180}{\tau} = \ln(.2) \quad \Rightarrow \quad \tau = 112(s)$$

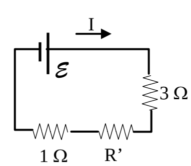


Fig. b1

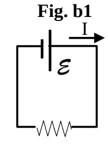


Fig. b2

Finally we obtain “R”:

$$RC = \tau \quad \Rightarrow \quad R = \frac{\tau}{C} = 1.87 \times 10^7(\Omega)$$

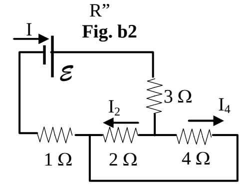


Fig. b3

(b) (i) The 2 and 4 ohm resistors are in parallel and can be replaced by R’ shown in Fig. b1:

$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{4} \quad \Rightarrow \quad R' = 1.33(\Omega)$$

The 1, 3 and 1.33 ohm resistors on Fig. b1 are in series and can be replaced by R” shown in Fig. b2:

R” = 1 + 3 + 1.33 = 5.33 (Ω)
(ii) The current in the 4 ohm resistor can be found since we know the power produced in it as heat:

$$P = I^2 R \quad \Rightarrow \quad I_4 = \sqrt{\frac{P}{R}} = 0.200(A)$$

The potential difference across the 4 ohm resistor is: $\Delta V_4 = I_4 R_4$ = 0.800 (V).
Since the 2 and 4 ohm resistors are in parallel, their potential differences must be the same. The current in the 2 ohm resistor is:

$$I_2 = \frac{\Delta V_2}{R_2} = \frac{.8}{2} = 0.400(A)$$

We can apply Kirchoff’s first rule at the junction point, shown in Fig. b3, between the 2, 3 and 4 ohm resistors to find the current “I” that passes through the battery:

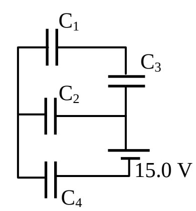
$$\sum I_{in} = \sum I_{out} \quad \Rightarrow \quad I = I_2 + I_4 = 0.600(A)$$

This 0.600 (A) current is also the current shown in Fig, b2. We apply Kirchoff’s Second Rule to this circuit we have:

$$Emf - IR'' = 0 \quad \Rightarrow \quad Emf = 3.20(V)$$

Physics 2402 Test 2 (11:30 Class) Fall 2007

1 The capacitors C_1 ($1.00\text{x}10^{-6}$ F), C_2 ($2.00\text{x}10^{-6}$ F), C_3 ($3.00\text{x}10^{-6}$ F) and C_4 ($4.00\text{x}10^{-6}$ F) are connected to a 15.0 (V) battery as shown in the diagram.
(a) Find the equivalent capacitance.
(b) Find the charge on C_4.
(c) Find the potential difference across C_3.



Solution:
1 (a) The capacitors C_1 and C_3 are in series and can be replaced by C’ shown in Fig.a:

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_3} \quad \Rightarrow \quad C' = 7.50 \times 10^{-7}(C)$$

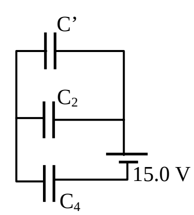


Fig. a

In Fig. a, the capacitors C’ and C_2 are in parallel and can be replaced by C” shown in Fig. b:

C” = C’ + C_2 = $2.75\text{x}10^{-6}$ (F).

The capacitors C_4 and C” in Fig. b are in series and can be replaced by C”’ shown in Fig. c:

$$\frac{1}{C'''} = \frac{1}{C_4} + \frac{1}{C''} \quad \Rightarrow \quad C''' = 1.63 \times 10^{-6}(C)$$

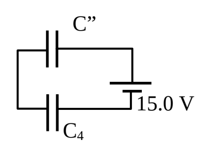


Fig. b

(b) The charge on C”’ is: Q”’ = C”’(ΔV) = $1.63\text{x}10^{-6}$(15) = $2.45\text{x}10^{-5}$(C). Since C”’ replaced C” and C_4 in series each of these capacitors has the same charge: Q_4 = Q” = $2.45\text{x}10^{-5}$(C).
(c) The potential difference across C” is:

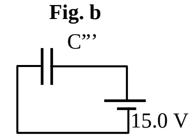


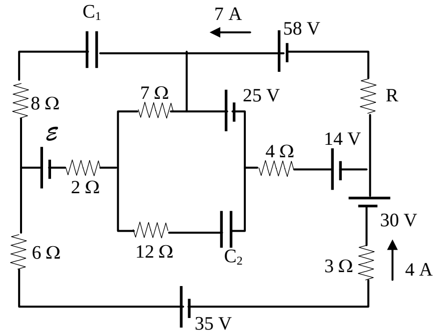
Fig. c

$$\Delta V'' = \frac{Q''}{C''} = 8.91(V)$$

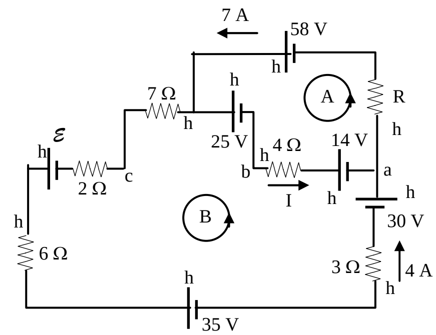
Since C” replaced C’ and C_2 in parallel each of these capacitors has the same potential difference as C” (8.91 V). The charge on C’ is: Q’ = C’ΔV’ = $7.50\text{x}10^{-7}$(8.91) = $6.69\text{x}10^{-6}$ (C).
Since C’ replaced C_1 and C_3 in series each of these capacitors have the same charges as C’ ($6.69\text{x}10^{-6}$ C). The potential difference across C_3 is:

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{6.69 \times 10^{-6}}{3 \times 10^{-6}} = 2.23(V)$$

2 In the circuit diagram the values of several resistances and battery *Emfs* are shown. Two currents along with their directions are also shown and both of the capacitors are fully charged. Assume all the given values have three significant figures.
(a) Find the unknown resistance "R".
(b) Find the unknown *Emf*, "$\mathcal{E}$".
(c) Find the energy stored in the capacitor C_2 which has a capacitance of $2.00\text{x}10^{-6}$ (F).



Solution:
(a) Since the capacitors are fully charged there are no currents in those portions of the circuit so the capacitors have not been included in the circuit diagram. The high potential sides of each battery and resistor have been marked. We can use Kirchoff's First rule at junction "a" to find the current "I". From a simple consideration of the magnitudes of the other currents at "a" we see that the current "I" must be directed into the junction:



$$\sum I_{in} = \sum I_{out} \quad \Rightarrow \quad I + 4 = 7 \quad \Rightarrow \quad I = 3(A)$$

We use Kirchoff's second law around loop "A" in a counterclockwise direction:

$$\sum_{loop\,A} \Delta V = 0 \quad \Rightarrow \quad -7R + 58 - 25 - 4(3) - 14 = 0 \quad \Rightarrow \quad R = 1.00(\Omega)$$

(b) We use Kirchoff's second law around loop "B" in a counterclockwise direction:

$$\sum_{loop\,B} \Delta V = 0 \quad \Rightarrow \quad -35 - 4(3) + 30 + 14 + 3(4) + 25 - 4(7) - 4(2) + Emf - 4(6) = 0 \quad \Rightarrow \quad Emf = 26.0(V)$$

(c) Since there is no current in the capacitor and 13 ohm resistor, the potential difference across this resistor is zero. Therefore the potential difference across the capacitor is $V_b - V_c$ where "b" and "c" are shown. This potential difference is:

$$V_b - V_c = \sum_{c \to b} \Delta V = +4(7) - 25 = 3.00(V)$$

The charge on C_2 and the energy stored are:

$$Q_2 = C_2 \Delta V_2 = 2 \times 10^{-6}(3) = 6.00 \times 10^{-6}(C)$$

$$U = \frac{1}{2} Q_2 \Delta V = 9.00 \times 10^{-6}(J)$$

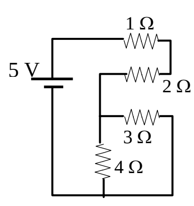


Fig. b

3 (a) A capacitor is connected directly to a 15.0 (V) battery until it becomes fully charged with a charge of $7.50\text{x}10^{-5}$ (C). It is then disconnected from the battery (without changing its charge) and is connected to a $2.40\text{x}10^{6}$ (Ω) resistor "R" and an open switch "S" as shown in Fig. a. The switch is then closed. How much time will elapse between the closing of the switch and the instant the charge on the capacitor is $3.00\text{x}10^{-5}$ (C)?
(b) Find the equivalent resistance of the circuit shown in Fig. b.
(c) To increase the power being produced as heat in the 2 ohm resistor in Fig. b to 2.70 (W), the 3 ohm resistor is replaced by a different resistor. What is the resistance of the new resistor?

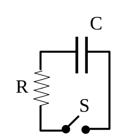


Fig. a

Solution:
(a) We can find the capacitance of the capacitor from the given information concerning its maximum charge when connected to the battery:

$$C = \frac{Q}{\Delta V} = \frac{7.5 \times 10^{-5}}{15} = 5.00 \times 10^{-6}(F)$$

The time constant of the RC discharge circuit is: t = RC = 12.0(s). The charge on the capacitor at any time "t" after the switch is closed is:

$$q(t)=Q_0 e^{\frac{-t}{\tau}} \Rightarrow e^{\frac{-t}{\tau}}=\frac{q(t)}{Q_0} \Rightarrow \frac{-t}{\tau}=\ln\left(\frac{q(t)}{Q_0}\right) \Rightarrow t=-\tau\ln\left(\frac{q(t)}{Q_0}\right)=-12\ln\left(\frac{3\times10^{-5}}{7.5\times10^{-5}}\right)=11.0(s)$$

(b) The 1 and 2 ohm resistors are in series and can be replaced by R' while the 3 and 4 ohm resistors are in series and can be replaced by R". These resistors are shown in Fig. 1:

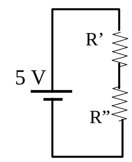


Fig. 1

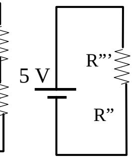


Fig. 2

$$R'=R_1+R_2=3.00(\Omega)$$

$$\frac{1}{R''}=\frac{1}{R_3}+\frac{1}{R_4} \Rightarrow R''=1.71(\Omega)$$

Finally the resistors R' and R" are in series and can be replaced by R"' shown in Fig. 2:
R"' = R' + R" = 4.71 (Ω).
(c) The current in the 2 ohm resistor is:

$$I_2=\sqrt{\frac{P_2}{R_2}}=1.16(A)$$

Since R' replaced the 1 and 2 ohm resistors in series the current in R' is also 1.16 (A).
We can apply Kirchoff's second rule around the loop shown in Fig. 3, where ΔV is the potential difference across R"" which is equivalent to the series capacitors of 4 ohms and R, the unknown resistance.
The equivalent resistance for R and the 4 ohm in parallel is shown in Fig. 4 and is:

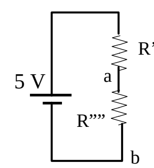


Fig. 3

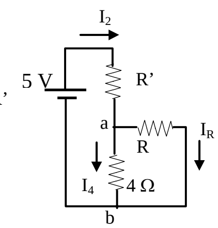


Fig. 4

$$\frac{1}{R''''}=\frac{1}{4}+\frac{1}{R} \Rightarrow R''''=\frac{4R}{4+R}$$

We apply Kirchoff's second rule around loop shown in Fig. 3. The current in the loop is 1.16 (A):

$$\sum\Delta V=0 \Rightarrow \quad =5-1.16(3)-1.16\left(\frac{4R}{4+R}\right) \Rightarrow R=1.95(\Omega)$$

Alternate Solution:

$$\sum\Delta V=0 \Rightarrow +5-1.16(3)-\Delta V \Rightarrow \Delta V=1.52(V)$$

The potential difference across the 4 ohm resistor is also 1.52 volts and the current through it is:

$$I_4=\frac{\Delta V}{R_4}=\frac{1.52}{4}=0.380(A)$$

We can apply Kirchoff's first rule at the junction "a" in Fig. 4 to find the current in R:

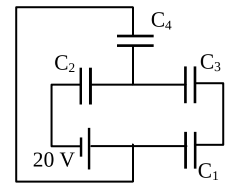
$$I_2=I_4+I_R \Rightarrow I_R=I_2-I_4=.780(A)$$

The resistance "R" is:

$$R=\frac{\Delta V_R}{I_R}=\frac{1.52}{.78}=1.95\ (\Omega)$$

Physics 2402 Test 2 (1:30 Class) Fall 2007

1 The capacitors C_1 ($1.00\text{x}10^{-6}$ F), C_2 ($2.00\text{x}10^{-6}$ F), C_3 ($3.00\text{x}10^{-6}$ F) and C_4 ($4.00\text{x}10^{-6}$ F) are connected to a 20.0 (V) battery as shown in the diagram.
(a) Find the equivalent capacitance.
(b) Find the charge on C_4.
(c) Find the energy stored in C_3.



Solution:
(a) The capacitors C_3 and C_1 are in series and can be replaced by C' shown in Fig. a.:

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_3} \quad \Rightarrow \quad C'=7.50\times10^{-7}(F)$$

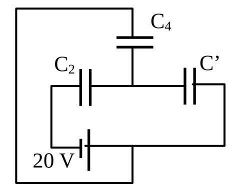


Fig. a

The capacitors C' and C_4 shown in Fig. a are in parallel and can be replaced by C" shown in Fig. b:

$C'' = C' + C_4 = 4.75\text{x}10^{-6}$ (F).
The capacitors C_2 and C" shown in Fig. b are in series and can be replaced by C"' shown in Fig. c.:

$$\frac{1}{C'''}=\frac{1}{C_2}+\frac{1}{C''} \quad \Rightarrow \quad C'''=1.41\times10^{-6}(F)$$

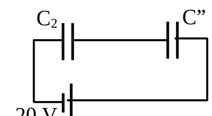


Fig. b

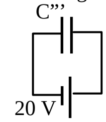


Fig.c

(b) The charge on C"' is: $Q''' = C'''\Delta V = 2.82\text{x}10^{-5}$ (C).
Since C"' replaced C" and C_2 in series each of these capacitors have the same charge as C"'. The potential difference across C" is:

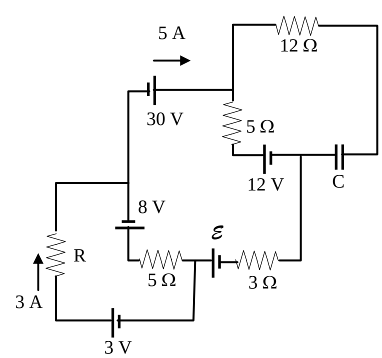
$$\Delta V''=\frac{Q''}{C''}=\frac{2.82\times10^{-5}}{4.75\times10^{-6}}=5.94(V)$$

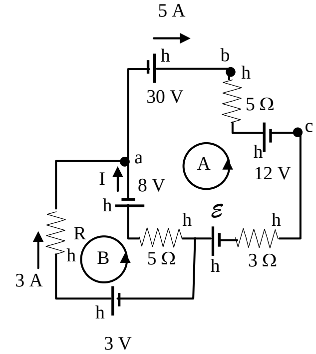
Since C" replaced C' and C_4 in parallel each of these capacitors have the same potential difference as C".
The charge on C_4 is:
$Q_4 = C_4\Delta V_4 = 4\text{x}10^{-6}(5.94) = 2.38\text{x}10^{-5}$ (C).

(c) The charge on C' is: $Q' = C'\Delta V' = 7.5\text{x}10^{-7}(5.94) = 4.46\text{x}10^{-6}$ (C). The charge on C_3 is the same as C' since C' replaced C_3 and C_1 in series. The energy stored in C_3 is:

$$U_3=\frac{1}{2}Q_3\Delta V_3=\frac{1}{2}\frac{Q_3^2}{C_3}=3.32\times10^{-6}(J)$$

2 In the circuit diagram the values of several resistances and battery *Emfs* are shown. Two currents along with their directions are also shown and the capacitor is fully charged. Assume all the given values have three significant figures.
(a) Find the unknown *Emf* "$\mathcal{E}$".
(b) Find the unknown resistance "R".
(c) Find the charge on the capacitor which has a capacitance of $3.00\text{x}10^{-6}$ (F).





Solution:
(a) Since the capacitor is fully charged, no current exists in that part of the circuit and it has not been shown. The unknown current "I" shown in the figure must be in the direction of the arrow to be consistent with Kirchoff's First Law at the junction "A":

$$\sum I_{in}=\sum I_{out} \quad \Rightarrow \quad 3+I=5 \quad \Rightarrow \quad I=2.00(A)$$

The high potential side of each resistor and battery is marked on the figure. We apply Kirchoff's Second Rule around loop "A" in the CCW direction:

$$\sum_{loop\ A} \Delta V = 0 \quad \Rightarrow \quad -\mathcal{E} + 5(3) + 12 + 5(5) - 30 + 8 + 2(5) = 0 \quad \Rightarrow \quad \mathcal{E} = 40.0(V)$$

(b) We apply the Second Rule around loop "B" in the CCW direction:

$$\sum_{loop\ B} \Delta V = 0 \quad \Rightarrow \quad -3 - 2(5) - 8 + 3R = 0 \quad \Rightarrow \quad R = 7.00(\Omega)$$

(c) The potential difference between the points "b" and "c" is:

$$V_b - V_c = \sum_{c \to b} \Delta V = +12 + 5(5) = 37.0(V)$$

The potential difference across the 12 ohm resistor is zero since the current through it is zero. Therefore, the potential difference across the capacitor is also 37.0 (V). The charge is: Q = CΔV = 1.11x10^{-4} (C).

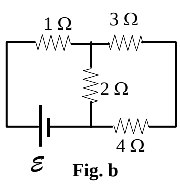


Fig. b

3 (a) The capacitor in Fig. a is initially uncharged. Exactly 2.00 (s) after the switch "S" is closed the potential difference ***across the resistor*** is 8.00 (V). Find the resistance "R".
(b) The heat produced in the resistors shown in Fig. b is used to keep four aircraft instruments at a constant temperature. The power being produced as heat in the three ohm resistor is 20.0 watts. (i) Find the equivalent resistance of the circuit and (ii) find the $\mathcal{E}$mf of the battery, "$\mathcal{E}$".

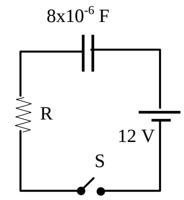


Fig. a

Solution:
(a) Since two seconds after the switch is closed the potential difference across the resistor is 8,00 (V), by Kirchoff's second rule applied around the circuit we have at this instant:

$$\sum \Delta V = 0 \quad \Rightarrow \quad \Delta V_R + \Delta V_C - \Delta V_{batt} \quad \Rightarrow \quad \Delta V_C = 12 - 8 = 4.00(V)$$

When the capacitor is fully charged there is no current and therefore no potential difference across the resistor. The final charge on the capacitor is: Q = CΔV = 8x10^{-6}(12) = 9.60x10^{-5} (C). Two seconds after the switch is closed the potential difference across the capacitor is:

$$\Delta V_c(2) = \frac{q(2)}{C} = \frac{Q\left(1 - e^{\frac{-2}{\tau}}\right)}{C} \quad \Rightarrow \quad 4 = \frac{9.6 \times 10^{-5}\left(1 - e^{\frac{-2}{\tau}}\right)}{8 \times 10^{-6}} \quad \Rightarrow \quad e^{\frac{-2}{\tau}} = .667 \quad \Rightarrow \quad \frac{-2}{\tau} = -.405 \quad \Rightarrow \quad \tau = 4.93$$

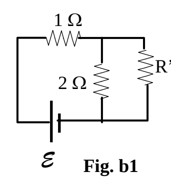


Fig. b1

The resistance is:

$$R = \frac{\tau}{C} = \frac{4.93}{8 \times 10^{-6}} = 6.16 \times 10^{5}(\Omega)$$

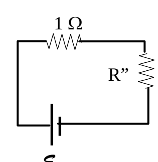


Fig. b2

(bi) The 3 and 4 ohm resistors are in series and can be replaced by R' in Fig. b1:
R' = 3 + 4 = 7 (Ω).
The resistor R' is in parallel with the 2 ohm resistor and can be replaced by R" in Fig. b2:

$$\frac{1}{R''} = \frac{1}{R'} + \frac{1}{2} \quad \Rightarrow \quad R'' = 1.56(\Omega)$$

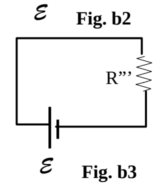


Fig. b3

The resistor R" and the one ohm resistor are in series and are replaced by R"' in Fig. b3:
R"' = R" + 1 = 2.56 (Ω).

(bii) The current in the 3 ohm resistor is:

$$I_3 = \sqrt{\frac{P_3}{3}} = 2.58(A)$$

Since this is also the current in R', the potential difference across R' is:

$\Delta V' = I'R' = I_3R' = 2.58(7) = 18.1$ (V)
The potential difference across R' is also the potential difference across R". The current in R" must be:

$$I'' = \frac{\Delta V''}{R''} = \frac{18.1}{1.56} = 11.6(A)$$

The current in R"' is the same as the current in R". We apply Kirchoff's second rule around the loop in Fig. b3:

$$\sum_{loop} \Delta V = 0 \quad \Rightarrow \quad Emf - I'''R''' = 0 \quad \Rightarrow \quad Emf = 11.6(2.56) = 29.7(V)$$