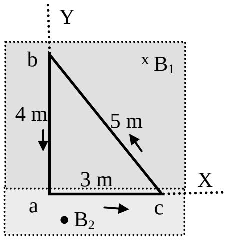
Physics 2402 Test #3 (8:30 Class) Fall 2006

1 The diagram shows a right angled triangular wire "abc" with a current of 2.00 (A) in the direction shown by the arrows. The direction of the positive "Z" axis is out of the page. The wire segments "ab" and "bc" are almost entirely in a uniform magnetic field B_1 having a magnitude of 0.800 (T) directed into the page. The segment "ac" is in a *different, non uniform* magnetic field B_2 directed out the page. This field depends on the "x" coordinate and is:

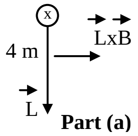
$$B_2 = 0.300x(T)$$



(a) Find the magnetic force exerted on "ab" in unit vector form.
(b) Find the magnetic force exerted on "bc" in unit vector form.
(c) Find the magnetic force exerted on "ac" in unit vector form.

Solution: (a) The magnitude of the force on "ab" is: F = ILBsin90 = 2(4)(.8) = 6.40(N). The direction of this force is determined by the vector cross product of L and B. This direction is found using the RHR and is in the positive X direction. The force is:

$$\vec{F}_{ab} = 6.40\hat{i}(N)$$



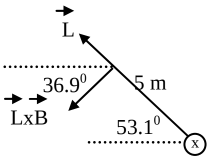
Part (a)

Or we can use:

$$\vec{L} = -4\hat{j}\ ;\ \vec{B} = -.8\hat{k}\ ;\ \vec{F}_{ab} = I\vec{L}\times\vec{B} = 2(4)(.8)\left(-\hat{j}\times-\hat{k}\right) = 6.40\hat{i}(N)$$

(b) From the dimensions of the triangle we find the angle shown in the figure. The magnitude of the force on "bc" is: F = ILBsin90 = 2(5)(.8) =8.00(N). The direction of this force is determined by the vector cross product of L and B. This direction is found using the RHR and is shown. The force is:

$$\vec{F}_{bc} = -8\cos 36.9\hat{i} - 8\sin 36.9\hat{j} = -6.40\hat{i} - 4.80\hat{j}(N)$$



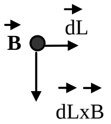
Part (b)

Or we can use:

$$\vec{L} = -5\cos 53.1\hat{i} + 5\sin 53.1\hat{j} = -3.00\hat{i} + 4.00\hat{j}\ ;\ \vec{B} = -.8\hat{k}\ ;\ \vec{F}_{bc} = I\vec{L}\times\vec{B} = 2\left(-3.00\hat{i} + 4.00\hat{j}\right)\times\left(-.8\hat{k}\right)$$

$$\vec{F}_{bc} = 4.80\left(\hat{i}\times\hat{k}\right) - 6.40\left(\hat{j}\times\hat{k}\right) = -6.40\hat{i} - 4.80\hat{j}(N)$$

(c) Since the field is not constant we must first find the force on a small length "dL" which is at a distance "x" from the origin. We replace "dL" by "dx". The magnitude of the force on "dL" is: dF = I(dL)B(x)sin90 = I(dx)(.3x)sin90 = 0.60x(N). The direction of this force is determined by the vector cross product of L and B. This direction is found using the RHR and is in the negative Y direction. The differential force is:



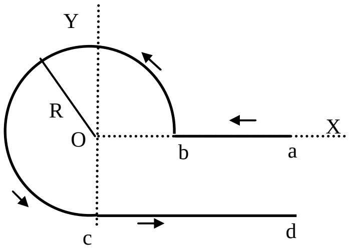
Part (c)

$$d\vec{F}_{ac} = -.60x\,dx\,\hat{j}$$

We have to integrate to get the total force on "ac":

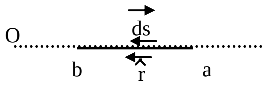
$$\vec{F}_{ac} = \int_0^3 d\vec{F}_{bc} = -\hat{j}(.60)\int_0^3 x\,dx = -\hat{j}(.60)\left[\frac{x^2}{2}\right]_0^3 = -2.70\hat{j}(N)$$

2 A wire "abcd" consists of two very long segments "ab" and "cd" and three quarters of a circle of radius "R". The wire has a current "I" in the direction shown by the arrows. The direction of the positive "Z" axis is out of the page.
(a) ***Derive***, using the ***Biot-Savart*** law, the magnetic field (in unit vector form) created at the origin by the current in the length "ab".
(b) ***Derive,*** using the ***Biot-Savart*** law, the magnetic field (in unit vector form) created at the origin by the current in the length "bc".
(c) Explain briefly why the magnetic field a perpendicular distance "R" from one end of a long, straight wire is half as large as the field a perpendicular distance "R" from the wire at the middle of the wire. Using this fact, determine the total field (in unit vector form) created by the current in the wire "abcd" at the origin.



Solution:(a) The magnitude of the differential field at the origin depends on the magnitude of the vector cross product:

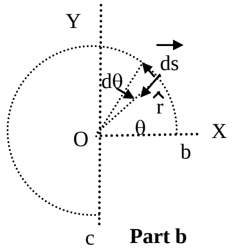
$$\left|d\vec{s}\times\hat{r}\right| = (ds)\sin 0 = 0$$



Part a

Since the magnitude of the differential field is zero the total field caused by the current in “ab” is zero.

(b) The important vectors are shown on the figure. The direction of the differential field at the origin caused by the current in the differential length “ds” is determined by the RHR and is out of the page in the +Z direction:



Part b

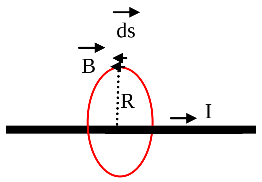
$$d\vec{s}\times\hat{r} = ds\hat{k} = rd\theta\hat{k}$$

The differential field is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{R^2} = \frac{\mu_0}{4\pi}\frac{\hat{k}IRd\theta}{R^2}$$

The total field is:

$$\vec{B}_{bc} = \frac{\hat{k}\mu_0 I}{4\pi R}\int_0^{2\pi/3} d\theta = \frac{\hat{k}\mu_0 I}{4\pi R}\left[\theta\right]_0^{2\pi/3} = \hat{k}\frac{3\mu_0 I}{8R}$$



Part c

(c) The field of a long straight wire forms circles about the wire as shown. The magnetic field at a perpendicular distance “R” from at its middle is:

$$B = \frac{\mu_0 I}{2\pi R}$$

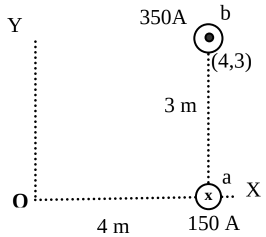
If the location is at the very center of the long wire, then by symmetry half of the total field comes from the wire to its left and the other half from the wire to its right. Therefore at one end of the long wire “cd” the field must be one half of the value computed above. Using the convenient RHR, we find that magnetic field of “cd” at the origin is out of the page. This field is:

$$\vec{B}_{cd} = \hat{k}\frac{\mu_0 I}{4\pi R}$$

The total field at the origin is:

$$\vec{B} = \hat{k}\left(\frac{\mu_0 I}{R}\right)\left(\frac{3}{8}+\frac{1}{4\pi}\right)$$

3 (a) The figure shows an end view of two very long, parallel wires “a” and “b”. The wire “a” (with a current of 150 A directed into the page) passes through the “XY” plane at (4,0) and wire “b” (with a current of 350 A directed out of the page) at (4,3) with the coordinates in meters. The positive “Z” axis is out of the page.



(a) Find the total magnetic field (in unit vector form) produced by both wires at the origin.

(b) A particle having a mass of 4.00x10^{-13} (kg) and a negative charge of -5.00x10^{-8} (C) is at the origin moving with a speed of 25.0 (m/s) in the negative X direction. Find the particle’s acceleration (in unit vector form) at this instant.

(c) A long straight wire of radius “a” has a uniform current I. ***Derive***, using Ampere’s Law, the magnitude of the magnetic field inside the wire at a distance “a/2” from its axis.

Solution: (a) From the given distances we find that the angle “θ” is 53.1^0. We first find the total field produced by the wires “a” and “b” at the origin. The directions of these fields are found using the convenient RHR and are shown in the figure. Since the wires are long, the magnitudes of the fields are:

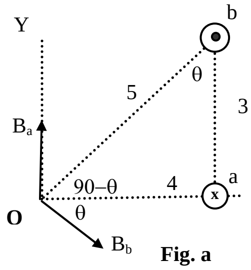


Fig. a

$$B_a = \frac{\mu_0 I_a}{2\pi r} = \frac{\mu_0(150)}{2\pi(4)} = 7.50\times10^{-6}(T)$$

$$B_b = \frac{\mu_0 I_b}{2\pi r} = \frac{\mu_0(350)}{2\pi(5)} = 1.40\times10^{-5}(T)$$

The total magnetic field is:

$$B_x = B_{ax} + B_{bx} = 0 + B_b\cos\theta = 1.40\times10^{-5}\cos 53.1 = 8.41\times10^{-6}(T)$$

$$B_y = B_{ay} + B_{by} = B_a - B_b\sin\theta = 7.50\times10^{-6} - 1.40\times10^{-5}\sin 53.1 = -3.70\times10^{-6}(T)$$

$$\vec{B} = 8.41\times10^{-6}\,\hat{i} - 3.70\times10^{-6}\,\hat{j}(T)$$

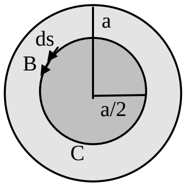


Fig. c

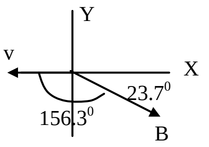
(b) The velocity of the charge and the magnetic force acting on it are:

$$\vec{v} = -25\hat{i}\left(\tfrac{m}{s}\right)$$

$$\vec{F} = q\vec{v}\times\vec{B} = \left(-5\times10^{-8}\right)\left(-25\hat{i}\right)\times\left(8.41\times10^{-6}\hat{i} - 3.70\times10^{-6}\hat{j}\right) = \left(-5\times10^{-8}\right)\left(-25\right)\left(-3.70\times10^{-6}\right)\left(\hat{i}\times\hat{j}\right) = -4.63\times10^{-12}\hat{k}\left(N\right)$$

The acceleration is:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-4.63\times10^{-12}}{4\times10^{-13}}\hat{k} = -11.6\hat{k}\left(\tfrac{m}{s^2}\right)$$



(b) Alternate Solution: The Magnetic field has a magnitude of 9.19x10^{-6} (T) and is at an angle of 23.7 degrees below the X axis as shown. These quantities are obtained from the components of the total field. The magnitude of the vector cross product of **B** and **v** is: vBsin156.3 = 9.23x10^{-5}. The direction of this cross product is obtained from the RHR and is in the positive Z direction. However, since the charge is negative, the magnetic force is in the negative Z direction. The magnitude of the force is:

$$F = |q||\vec{v}\times\vec{B}| = 5\times10^{-8}\left(9.23\times10^{-5}\right) = 4.62\times10^{-12}\left(N\right)$$

$$\vec{F} = -4.62\times10^{-12}\hat{k}\left(N\right)$$

(c) Fig. c shows the wire with the current coming out of the page. Since the magnetic field forms circles about the axis of the wire we choose a circular path "C" of radius "a/2" to find the field. On this path both the vector **ds** and **B** are parallel. The current density and the current inside the path "C" are:

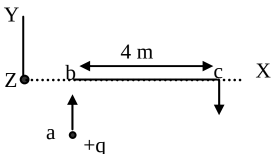
$$J = \frac{I}{A} = \frac{I}{\pi a^2} \quad and \quad I_{in} = J\left(\pi\left(\frac{a}{2}\right)^2\right) = \frac{I}{4}$$

We use Ampere's Law:

$$\int_C \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \quad \Rightarrow \quad \int_C B\left(\cos 0\right)ds = \frac{\mu_0 I}{4}$$

$$B\int_C ds = \frac{\mu_0 I}{4} \quad \Rightarrow \quad B\left(2\pi\left(\frac{a}{2}\right)\right) = \frac{\mu_0 I}{4} \quad \Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}$$

Physics 2402 Test #3 (11:30 Class) Fall 2006



1 In the diagram the direction of the positive Z axis is out of the page. Below the line "bc" there is an unknown, uniform electric field and a uniform magnetic field:

$$\vec{B}_1 = -0.200\hat{k}\left(T\right)$$

Above the line "bc" there is a different uniform magnetic field B$_2$ that is perpendicular to the XY plane *but no electric field*. A particle having a mass of 3.00x10^{-6}(kg) and a positive charge of +9.00x10^{-6} (C) is at "a" is moving parallel to the "Y" axis towards "b" with a constant speed of 4.00 (m/s). After passing through "b", it subsequently moves along a semicircular path to "c" where it is moving in the direction of the arrow. The distance between "b" and "c" is 4.00 (m).

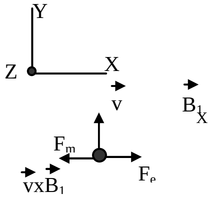
(a) Find the electric field in unit vector form.

(b) Find the magnetic field $\vec{B}_2$ in unit vector form.

(c) Find the acceleration of the charge in unit vector form ***just after*** it passed "c" and moved into the region having both electric and magnetic fields.

Solution: (a) The magnitude of the magnetic force is:

$$F_m = qvB_1\sin 90 = qvB_1$$



Part a

Using the RHR we find the direction of this force. As shown it is in the negative "X" direction. The magnitude of the electric force is qE and must be in the positive direction as shown so that the acceleration will be zero. Since the charge is positive, the electric field and force are in the same direction:

$$\sum F_x = ma_x = 0 \quad \Rightarrow \quad qE - qvB_1 = 0 \quad \Rightarrow \quad E = vB_1 = 4(.2) = .800\left({}^{N}\!/_{C}\right)$$

$$\vec{E} = E\hat{i} = .800\hat{i}\left({}^{N}\!/_{C}\right)$$

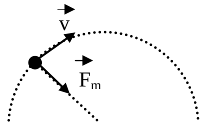
Alternate solution:

$$\vec{F}_m = q\vec{v}\times\vec{B} = q\left(4\hat{j}\right)\times\left(-.2\hat{k}\right) = -.8q\left(\hat{j}\times\hat{k}\right) = -.8q\hat{i}$$

$$\vec{F}_e = q\vec{E}$$

For no acceleration:

$$\sum \vec{F} = m\vec{a} = 0 \quad \Rightarrow \quad \vec{F}_m + \vec{F}_e = 0 \quad \Rightarrow -.8q\hat{i} + q\vec{E} = 0 \Rightarrow \vec{E} = .800\hat{i}$$



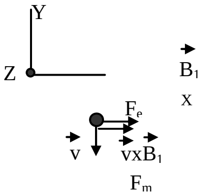
Part b

(b) Since the magnetic field above "bc" is perpendicular to the XY plane the direction of the magnetic force on the moving charge will cause it to move in a semicircular path from "b" to "c" having a radius of 1.50 (m). By applying the RHR at the location shown, the magnetic field must be out of the page in the +Z direction to make the magnetic force be in the direction shown. We apply Newton's II Law to the motion of the charge:

$$F_m = qvB_2 sn90$$

$$\sum F_{rad} = \frac{mv^2}{r} \Rightarrow qvB_2 = \frac{mv^2}{r} \Rightarrow B_2 = \frac{mv}{qr} = \frac{3\times10^{-6}(4)}{9\times10^{-6}(2)} = 0.667(T)$$

$$\vec{B}_2 = .667\hat{k}(T)$$



Part c

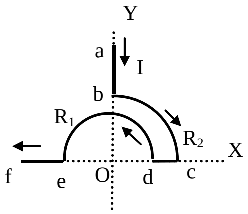
(c) The direction of the vector cross product of the velocity and the magnetic field is in the positive "X" direction as is the electric field. The acceleration of the charge is:

$$a_x = \frac{\sum F_x}{m} = \frac{q(E + vB_1)}{m} = \frac{9\times10^{-6}(.8 + 4(.2))}{3\times10^{-6}} = 4.80\left({}^{m}\!/_{s^2}\right) \quad \Rightarrow \vec{a} = 4.80\hat{i}\left({}^{m}\!/_{s^2}\right)$$

or,

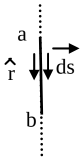
$$\vec{a} = \frac{\sum\vec{F}}{m} = \frac{q\vec{E} + q\vec{v}\times\vec{B}_1}{m} = \left(\frac{q}{m}\right)\left(\vec{E} + \vec{v}\times\vec{B}\right) = 3\left(.8\hat{i} + \left(\left(-4\hat{j}\right)\times\left(-.2\hat{k}\right)\right)\right) = 3\left(.8\hat{i} + .8\hat{i}\right) = 4.80\hat{i}\left({}^{m}\!/_{s^2}\right)$$

2 A wire "abcdef" has two very long straight sections "ab" and "ef". The section "bc" is part of a circle of radius "R_2" and the section "de" is part of a circle of radius "R_1". The direction of the positive "Z" axis is out of the page and the wire has a current "I" in the direction shown by the arrows.
(a) ***Derive***, using the Biot Savart Law, the magnetic field (in unit vector form) at the origin caused by the current in the section "ab".
(b) ***Derive***, using the Biot Savart Law, the magnetic field (in unit vector form) at the origin caused by the current in the section "de".
(c) Find the total magnetic field (in unit vector form) caused by the current in all of the sections.



Solution: (a) The magnitude of the differential field at the origin depends on the magnitude of the vector cross product:

$$\left|d\vec{s}\times\hat{r}\right| = (ds)\sin 0 = 0$$



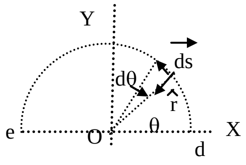
Part a

Since the magnitude of the differential field is zero the total field caused by the current in "ab" is zero.
(b) The important vectors are shown on the figure. The direction of the differential field at the origin caused by the current in the differential length "ds" is determined by the RHR and is out of the page in the +Z direction:

$$d\vec{s}\times\hat{r} = ds\hat{k} = R_1 d\theta\hat{k}$$

The differential field is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{R_1^2} = \frac{\mu_0}{4\pi}\frac{\hat{k}IR_1 d\theta}{R_1^2}$$



Part b

The field is:

$$\vec{B}_{de}=\frac{\hat{k}\mu_0 I}{4\pi R_1}\int_0^{\pi}d\theta=\frac{\hat{k}\mu_0 I}{4\pi\, R_1}[\theta]_0^{\pi}=\frac{\mu_0 I\hat{k}}{4R_1}$$

(c) For the path “bc” we can obtain the field using the result from (b) with the limits of the integral changed. However the direction of this field is in the negative “Z” direction since the direction of the vector cross product is reversed. The fields of “dc” and “ef” are zero for the same reason as in (a). The total field is:

$$\vec{B}_{total}=\vec{B}_{de}+\vec{B}_{bc}=\frac{\mu_0 I\hat{k}}{4R_1}-\frac{\hat{k}\mu_0 I}{4\pi R_2}\int_0^{\pi/2}d\theta=\frac{\mu_0 I\hat{k}}{4R_1}-\frac{\hat{k}\mu_0 I}{4\pi\, R_1}[\theta]_0^{\pi/2}=\frac{\mu_0 I\hat{k}}{4}\left(\frac{1}{R_1}-\frac{1}{2R_2}\right)$$

3 (a) The figure shows an end view of three very long, parallel wires “a”, “b” and “c”. The currents and their directions are shown on the diagram. Distances between the centers of the wires are shown.
(i) Find the total magnetic field (in unit vector form) created by the wires “a” and “b” at the location of wire “c”.
(ii)Find the total magnetic force (in unit vector form) that the wires “a” and “b” exert on a 20.0 (m) length of the wire “c”.

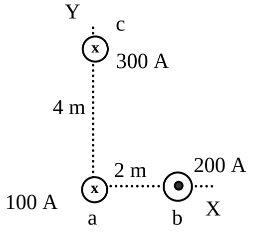


Fig. a

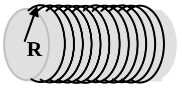


Fig. b

(b) This part of the problem is not related to part (a). Figure b shows a long coil having “n” turns of wire per unit length. Each turn has the same current “I” and a radius “R”. ***Derive***, using Ampere’s Law, the magnitude of the magnetic field inside the coil at a location not too close to either end.

Solution: (a) From the given distances we find that the angle “θ” is 26.6^0. We first find the total field produced by the wires “a” and “b” at the location of ‘c”. The directions of these fields are found using the convenient RHR and are shown in the figure. Since the wires are long, the magnitudes of the fields are:

$$B_a=\frac{\mu_0 I_a}{2\pi r}=\frac{\mu_0(100)}{2\pi(4)}=5.00\times10^{-6}(T)$$

$$B_b=\frac{\mu_0 I_b}{2\pi r}=\frac{\mu_0(200)}{2\pi(4.47)}=8.95\times10^{-6}(T)$$

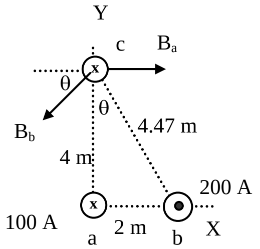


Fig. a

The total magnetic field is:

$$B_x=B_{ax}+B_{bx}=B_a-B_b\cos\theta=5\times10^{-6}-8.95\times10^{-6}\cos26.6=-3.00\times10^{-6}(T)$$

$$B_y=B_{ay}+B_{by}=-B_b\sin\theta=-8.95\times10^{-6}\sin26.6=-4.00\times10^{-6}(T)$$

$$\vec{B}=-3.00\times10^{-6}\hat{i}-4.00\times10^{-6}\hat{j}(T)$$

For a 20 (m) length of the wire “c” the vector length is: $\vec{L}=-20\hat{k}(m)$
The force exerted on this length by the total field is:

$$\vec{F}=I_c\vec{L}\times\vec{B}=300\left(-20\hat{k}\right)\times\left(-3\times10^{-6}\hat{i}-4\times10^{-6}\hat{j}\right)=300\left(20\hat{k}\right)\times\left(3\times10^{-6}\hat{i}+4\times10^{-6}\hat{j}\right)$$

$$\vec{F}=.018\left(\hat{k}\times\hat{i}\right)+.024\left(\hat{k}\times\hat{j}\right)=-.024\hat{i}+.018\hat{j}(T)$$

(b) The figure sows a side view of the long coil. As discussed in class the magnetic field is parallel to the axis if we are not too close to either end of the coil. We choose a rectangular path “abcd” for “C”. The magnetic field outside the long coil is assumed to be zero and on “bc” and “da” the field is perpendicular to the path so that the scalar product of ‘ds” and “B” is zero. We only have to evaluate the line integral along “ab” where “B” and “ds” are parallel. The length “ab” is “D” so that the number of turns passing through the path “C’ is “nD”. The total current inside the path is “nDI”. We apply Ampere’s Law:

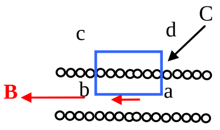


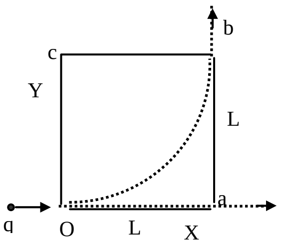
Fig. b

$$\int_{ab}\vec{B}\cdot d\vec{s}=\mu_0\left(I_{in}\right)\;\Rightarrow\;\int_{ab}B\cos0ds=\mu_0(nDI)\Rightarrow B\int_{ab}ds=\mu_0 nDI$$

$$B(D)=\mu_0 nDI\quad\Rightarrow B=\mu_0 nI$$

Physics 2402   Test #3 (1:30 Class)   Fall 2006

1 In the diagram, the direction of the positive Z axis is out of the page. A cubic box has sides of unknown length “L”. The figure shows a cross section of the box “Oabc” with the origin at “O”. Three small holes are located at the corners “O”, “a” and “b”. Inside the box there is a uniform magnetic field of 0.320(T) that is perpendicular to the “XY” plane. A very small particle with a ***negative*** charge of -5.00×10^{-6} (C) and a mass of 8.00×10^{-8} (kg) is moving along the negative “X” axis towards the origin with a speed of 25.0 (m/s). After it enters the box at “O”, it subsequently leaves the box at “b” where it is moving parallel to the “Y” axis with the same speed. Now an unknown, electric field that is in the “XY” plane is applied in *addition to the magnetic field*. A second, identical particle initially moving along the negative “X’ axis with the same speed of 25.0 (m/s) enters the box at “O” and then moves with a constant speed along the “X” axis leaving the box at “a”.



(a) Determine the direction of the magnetic field and express the field in unit vector form.
(b) Find the unknown length “L”.
(c) Find the electric field in unit vector form.

Solution: (a) The charge will follow a path which is one quarter of a circle of radius “L” with its center at “c”. The direction of the magnetic field is shown in Fig. c. Since the charge is negative, the force is in the opposite direction. By applying the RHR, the direction of the magnetic field must be out of the page to give the correct direction for the cross product:

$\vec{B} = .320\hat{k}(T)$

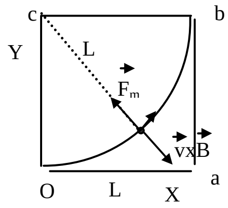


Fig b

(b) The charge will follow a path which is one quarter of a circle of radius “L” with its center at “c”. The direction of the vector cross product of the velocity and the magnetic field is shown. Since the charge is negative, the force is in the opposite direction. The radial force acting on the charge is the magnetic force which is always perpendicular to the path. We apply Newton’s II law to the motion:

$$\sum F_{rad} = \frac{mv^2}{L} \quad \Rightarrow \quad |q|vB = \frac{mv^2}{L} \Rightarrow L = \frac{mv}{|q|B} = \frac{8\times10^{-8}(25)}{5\times10^{-6}(.32)} = 1.25(m)$$

(c) For the charge to move in a straight line from “O” to “a” at constant speed the acceleration must be zero. The magnetic force is:

$\vec{F}_m = q\vec{v}\times\vec{B} = q(25\hat{i})\times(.32\hat{k}) = -8q\hat{j}$

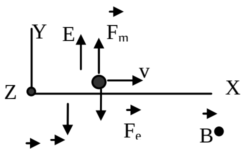
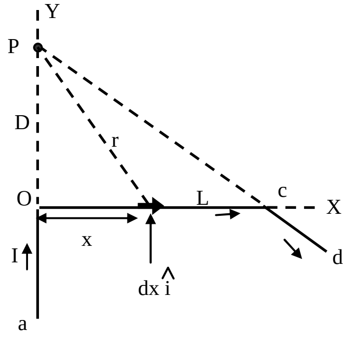


Fig b

For no acceleration we have:

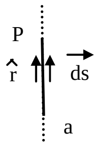
$$\sum\vec{F} = 0 \Rightarrow \vec{F}_m + \vec{F}_e = 0 \Rightarrow \quad -8q\hat{j} + q\vec{E} = 0 \quad \Rightarrow \quad \vec{E} = 8\hat{j}(N/C)$$

2 The wire “aOcd” has two very long segments “aO” and “cd” and a segment “Oc” that has a length “L”. The wire has a current “I” in the direction of the arrows. The location “P” is on the “Y” axis a distance “D” above the origin. The direction of the positive Z axis is out of the page.
(a) ***Derive***, using the Biot-Savart Law, the magnetic field (in unit vector form) at “P” caused by the current in “aO”.
(b) The figure shows a differential length vector “dx **i**” on the segment “Oc”. This differential length vector is a distance “x” from the origin. ***Derive***, using the Biot–Savart Law, the differential magnetic field d**B** at “P”, in unit vector form, caused by the current in the differential length only.
(c) Find the total field (in unit vector form) at “P” caused by the current in the segment “Oc”. You do not need to evaluate the integral, however, you should simplify your result as much as possible.



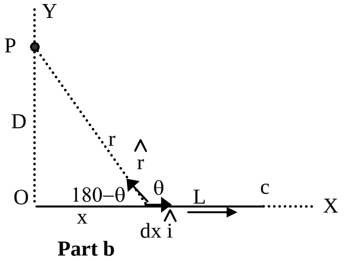
Solution:
2 (a) The magnitude of the differential field at “P” depends on the magnitude of the vector cross product:

$|d\vec{s}\times\hat{r}| = (ds)\sin 0 = 0$



Part a

Since the magnitude of the differential field is zero the total field caused by the current in “aO” is zero. Also on the segment “cd” the above cross product is also zero since the angle is 180 degrees and its sine is zero.
(b) The figure shows the vectors we need to apply the Biot-Savart law to obtain the differential field at “P”.
The direction of the vector cross product of dx**i** and the unit vector **r** is found by the RHR to be in the positive Z direction. The differential field is:



Part b

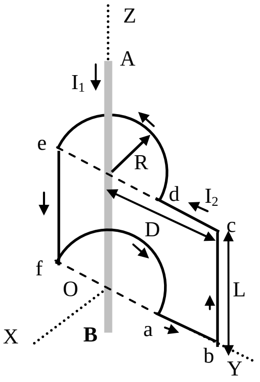
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(dx\hat{i}\right)\times\hat{r}}{r^2} = \hat{k}\frac{\mu_0 I dx\sin\theta}{4\pi r^2}$$

$$d\vec{B} = \hat{k}\frac{\mu_0 I dx\sin(180-\theta)}{4\pi r^2} = \hat{k}\frac{\mu_0 I dx}{4\pi r^2}\left(\frac{D}{r}\right) = \hat{k}\frac{\mu_0 D I dx}{4\pi\left(x^2+D^2\right)^{3/2}}$$

(c) The total field is found by integrating the differential field for x = 0 to x = L.

$$\vec{B}_{Oc} = \hat{k}\frac{\mu_0 D I}{4\pi}\int_0^L \frac{dx}{\left(x^2+D^2\right)^{3/2}}$$

3 (a) A long, thin, straight wire “AB” is located along the Z axis and has a current I_1 in the negative direction as shown. ***Derive***, using Ampere’s Law, the magnitude of the magnetic field at a perpendicular distance “r” from the wire, but not too close to either end.
For the following parts, the current in the wire “AB” is 5.00 (A).
(b) A second wire “abcdefa” has a current I_2 of 8.00 (A) in the direction shown by the arrows. The side “bc” has a length “L” of 0.200 (m) and is parallel to the wire “AB”. The distance “D” between the wire “AB” and the wire “bc” is 0.300 (m). Find the force exerted by the wire “AB” on the wire segment “bc” in unit vector form.
(c) The wire segment “de” is a semicircle having a radius “R” of 0 .150 (m). This semicircle is parallel to the “XY” plane and its center is on the “Z” axis. Find the force exerted on the segment “de” by the long wire “AB”.
(d) Find the force (in unit vector form) that the long wire exerts on the segment “ab”. Note that “a” is a distance of 0.150 (m) from the long wire “AB” and “b” is 0.300 (m) from it.



Solution: (a) The field is circular about the wire and is in the direction shown by the arrow. We choose a circular path “C” of radius “r” on which the differential displacement “ds” and the magnetic field are parallel. We use Ampere’s law noting that the current passing through “C” is “I”:

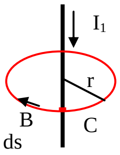


Fig. a

$$\int_C \vec{B}\cdot d\vec{s} = \mu_0 I \Rightarrow \int_C B\cos 0 ds = \mu_0 I \Rightarrow B\int_C ds = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I \ \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(b) The direction of the magnetic field of the wire “AB” at the location of “bc” is found by our convenient RHR and is shown in Fig. b. The direction of the vector cross product of L and B is in the positive “Y” direction by the RHR. The force on “cb: is:

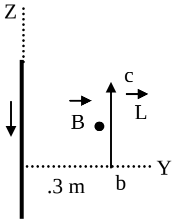


Fig. b

$$F_{bc} = I_2 LB\sin 90 = I_2 L\left(\frac{\mu_0 I_1}{2\pi D}\right) = \frac{\mu_0 I_1 I_2 L}{2\pi D} \Rightarrow \quad \vec{F}_{bc} = \frac{\mu_0 I_1 I_2 L}{2\pi D}\hat{j} = 5.33\times10^{-6}(N)$$

or

$$\vec{F}_{bc} = I_2\vec{L}\times\vec{B} = I_2\left(L\hat{k}\right)\times\left(\frac{\mu_0 I_1}{2\pi D}\hat{i}\right) = \frac{\mu_0 I_1 I_2 L}{2\pi D}\left(\hat{k}\times\hat{i}\right) = \frac{\mu_0 I_1 I_2 L}{2\pi D}\hat{j} = 5.33\times10^{-6}(N)$$

(c) At any point on the path “de”, the magnetic field caused by wire “AB and the differential vector “dL” are in the opposite direction. These vectors are shown in Fig. c. The magnitude of the force on “dL” is:

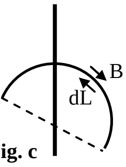


Fig. c

$$dF = I_2(dL)B\sin(180) = 0$$

Therefore the total force on “de” is zero.
(d) The field of wire “AB” changes along the length of wire “ab” so we must find the force on a differential length “dL”. By the RHR the direction of the cross product “dL x B” is in the negative “Z” direction. We can replace “dL” by “dy”. The force on a length “dy” of the wire “ab” is:

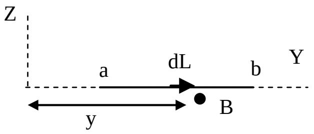


Fig. d

$$d\vec{F} = I_2(dy)B\sin 90\left(-\hat{k}\right) = I_2(dy)\left(\frac{\mu_0 I_1}{2\pi y}\right)\left(-\hat{k}\right)$$

$$\vec{F} = \int d\vec{F} = \frac{-\hat{k}\mu_0 I_1 I_2}{2\pi}\int_{.15}^{.300}\frac{dy}{y} = \frac{-\hat{k}\mu_0 I_1 I_2}{2\pi}\left[\ln r\right]_{.15}^{.30} = -5.55\times 10^{-6}\,\hat{k}(N)$$