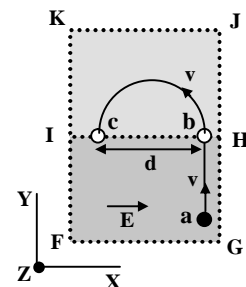


Physics 2402 Test #3 (8:30 Class) Fall 2004

1 In the figure, the directions of the axes are shown. The “Z” axis is out of the page. In the region “FGHI” there is a uniform electric field having a magnitude of 0.160 (N/C) in the positive “X” direction and an unknown magnetic field which is parallel to the “Z” axis. In the region “IHJK” there is no electric field but there is an unknown magnetic field which is parallel to the “Z” axis. This magnetic field is *not the same* as the other magnetic field. A negatively charged particle has a charge of -5.00×10^{-6} (C) and a mass of 6.00×10^{-6} (kg). Initially the particle is at “a” moving with a speed of 2.00 (m/s) in the positive “Y” direction. The particle continues moving with this constant speed in the “+Y” direction until it reaches “b”. At “b” it enters the region “IHJK” and later it emerges from this region at “c” moving in the negative “Y” direction with the same constant speed of 2.00 (m/s). The distance “d” between “b” and “c” is 16.0 (m) and the gravitational force is neglected.



- (a) Find the magnetic field (in unit vector form) in the region “FGHI”.
- (b) Find the magnetic field (in unit vector form) in the region “IHJK”.
- (c) Prove that the time it takes for the particle to move from “b” to “c” is not dependent on the radius of its path.

Solution:

1 Fig. a shows the particle when it is in the region “FGHI”. The electrical force acting on the particle is:

$$\vec{F}_e = q\vec{E} = qE\hat{i} \text{ (N/C)}$$

Since “q” is negative, this force is in the negative “X” direction as shown. Next we determine the direction of the magnetic field. Since the particle moves with a constant velocity its acceleration is zero and therefore the net force must be zero. From the diagram it should be clear that the magnetic force must be in the positive “X” direction. The magnetic force on the particle is:

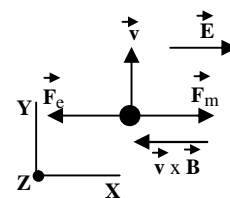


Fig. a

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

Since the charge is negative the magnetic force is in the direction opposite to the vector cross product of the velocity and magnetic field. Therefore, the vector cross product must be in the negative “X” direction as shown in the figure. By using the RHR the magnetic field must be in the negative “Z” direction to make the vector cross product be in the negative “X” direction. For no acceleration we have:

$$\sum \vec{F} = 0 \Rightarrow \vec{F}_e + \vec{F}_m = 0 \Rightarrow qE\hat{i} + qv\hat{j} \times B(-\hat{k}) = 0$$

$$E\hat{i} - vB\hat{i} = 0 \Rightarrow B = \frac{E}{v} = .0800(T)$$

$$\vec{B} = -.0800\hat{k}(T)$$

(b) Since the velocity in region “IHJK” is perpendicular to the magnetic field, the particle must move along a semicircular path from “b” to “c”. The diameter of the semicircular path is 16.0 (m) and the radius is 8.00 (m). The direction of the magnetic force must be radial as shown in the figure. Since the charge is negative, the direction of the magnetic force is opposite to the direction of the vector cross product of the velocity and the magnetic field. By applying the RHR we find that the magnetic field must be out of the page in the positive “Z” direction to give the correct direction for the cross product. We apply Newton’s second law to the circular motion of the charge:

$$\sum F_{radial} = m \frac{v^2}{R} \Rightarrow |q|vB \sin 90 = \frac{mv^2}{R}$$

$$B = \frac{mv}{|q|R} = \frac{6 \times 10^{-6}(2)}{5 \times 10^{-6}(8)} = 0.300(T)$$

$$\vec{B} = .300\hat{k}(T)$$

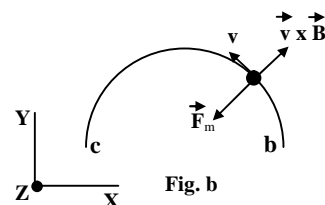


Fig. b

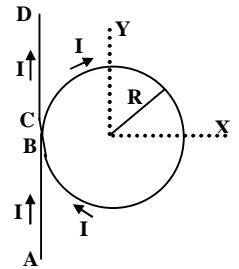
(c) From part (b) the speed of the particle which we can use to find the time to go from “b” to “c”:

$$v = \frac{|q|RB}{m}$$

$$T = \frac{pR}{v} = \frac{pRm}{|q|RB} = \frac{pm}{|q|B}$$

The time does not depend on the radius “R”.

2 In the figure, “AB” is a long straight wire carrying a current “I” in the direction shown by the arrow. At “B” the current passes through a circular wire loop of radius “R” in the direction shown by the arrows. The loop’s center is at the origin. After leaving the loop at “C”, the current moves along the long straight section of wire “CD” in the direction shown.



(a) **Derive**, using the **Biot-Savart Law**, the magnetic field (in unit vector form) at the origin produced by the current in the circular wire loop.

(b) The separate straight line sections “AB” and “AC” can be considered to be a single, long, straight wire section “AD” carrying a current “I”. Derive, using **Ampere’s Law**, the magnetic field (in unit vector form) at the origin produced by the current in the wire “AD”.

Solution:

2 (a) The directions of the vectors **ds** and **r** are shown in Fig. a. The direction of the cross product of these vectors is found using the RHR and is into the page in the negative Z direction. The magnitude of ds is: ds = R dθ. The magnetic field at the origin produced by the current in “BC” is:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{-\hat{k} R dq}{R^2} = -\frac{\mu_0 I \hat{k}}{4\pi R} \int_0^{2\pi} dq$$

$$\vec{B} = -\frac{\mu_0 I \hat{k}}{4\pi R} [q]_0^{2\pi} = -\frac{\mu_0 I \hat{k}}{2R}$$

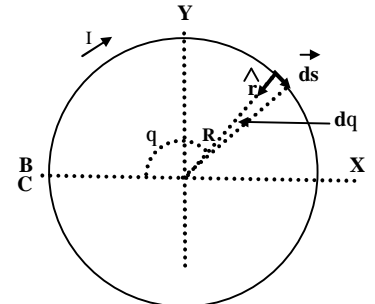


Fig. a

The unit vector “**k**” and the radius “R” are constants and can be removed from the integral. The limits for the integration are zero to 2π we assume that the wire loop BC forms a complete circle.

(b) The magnetic field forms circles about the long wire which is shown in a side view in Fig. b1 and looking into the wire from end “A” in Fig. b2. In Fig. b2 the convenient right hand rule is used to find the direction of the field. We curl our fingers around the wire with our thumb extended in the direction of the current. The field direction is shown in the figure and at the origin its direction is seen to be the negative “Z” direction. The direction of the field at the origin is also shown in Fig. b1 and is into the page (the negative Z direction.)

We apply Ampere’s Law using the circular path “C” of radius “R”. Along this path the vectors **ds** and **B** are parallel. The current inside this path is “I”.

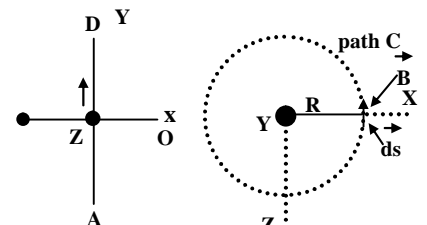


Fig. b1

Fig. b2

Ampere’s Law gives:

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 I \Rightarrow \int_C B \cos 0 ds = \mu_0 I$$

$$B \int_C ds = \mu_0 I \Rightarrow B(2\pi R) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi R} (-\hat{k})$$

In the above equations, B is a constant along the path and has been removed from the integral. The integral of “ ds ” over the circle is the circumference ($2\pi R$) of the circle.

3 The figure shows a long straight wire that is perpendicular to the page and passes through the origin. It has a current “ I ” directed out of the page in the positive “ Z ” direction. The magnitude of the magnetic field caused by this current a perpendicular distance “ r ” from the wire is:

$B = \mu_0 I / (2\pi r)$. The wire loop “ $abcd$ ” has a current “ I_{loop} ” in the direction shown by the arrows.

The curved portions of the loop are quarter circles of radii “ R_1 ” and “ R_2 ”.

(a) A small length “ dL ” is located on the curved segment “ bc ”. Find the magnitude of the magnetic force “ dF ” exerted on the length “ dL ” by the current in the long straight wire. What is the total magnetic force on “ bc ”?

(b) Find (in unit vector form) the force “ dF ” exerted, by the current in the long straight wire, on the small length “ dL ” which is on “ ab ” a distance “ x ” from the origin. The length of “ dL ” can also be expressed as “ dx ”.

(c) Find the total magnetic force (in unit vector form) exerted on “ ab ”.

Solution:

3 (a) The magnetic fields of a long straight wire form circles around the wire. By using the convenient RHR the direction of the field is found and is shown in Fig. a. The direction of the vector length “ dL ” is determined by the direction of the current. The direction is shown in the figure. Since the angle between the vectors “ B ” and “ dL ” is zero the magnitude of “ dF ” is: $dF = I(dL)B\sin 0 = 0$. Since “ dF ” is zero the total force on the segments “ bc ” and “ da ” are zero.

(b) The direction of the differential length vector “ dL ” on the wire “ ab ” is the positive “ X ” direction and is shown in Fig. b. The direction of the magnetic field of the long wire at the location of “ dF ” is found using the convenient RHR. The magnetic field is in the positive “ Y ” direction. The direction of the cross product “ $dL \times B$ ” is in the positive “ Z ” direction. The magnitude of the differential magnetic force on “ dL ” which is a distance “ x ” from the long wire is:

$$dF = I_{loop} (dL) B \sin 90 = I_{loop} (dx) \left(\frac{\mu_0 I}{2\pi x} \right) = \frac{\mu_0 I I_{loop} dx}{2\pi x}$$

In unit vector form:

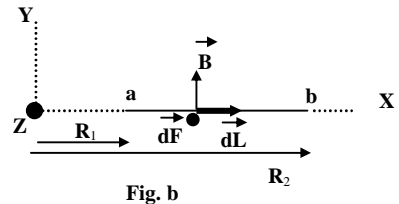
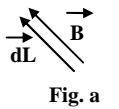
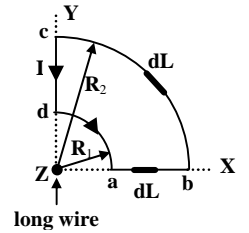
$$d\vec{F} = \frac{\hat{k} \mu_0 I I_{loop} dx}{2\pi x}$$

Alternate solution:

$$d\vec{F} = I_{loop} dx \hat{i} \times \frac{\mu_0 I \hat{j}}{2\pi x} = \frac{\mu_0 I I_{loop} dx}{2\pi x} (\hat{i} \times \hat{j}) = \frac{\hat{k} \mu_0 I I_{loop} dx}{2\pi x}$$

$$\vec{F} = \int_{R_1}^{R_2} \frac{\hat{k} \mu_0 I I_{loop} dx}{2\pi x} = \frac{\hat{k} \mu_0 I I_{loop}}{2\pi} \int_{R_1}^{R_2} \frac{dx}{x} = \frac{\hat{k} \mu_0 I I_{loop}}{2\pi} [\ln x]_{R_1}^{R_2} = \frac{\hat{k} \mu_0 I I_{loop}}{2\pi} \ln \frac{R_2}{R_1}$$

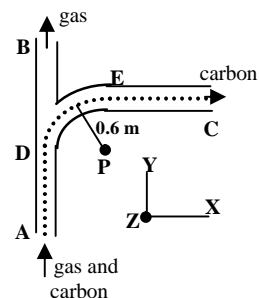
(d) The forces on the curved sides are zero. For the wire “ cd ” the direction of “ dL ” is in the negative “ Y ” direction and the vector cross product of “ dL ” and “ B ” is in the negative “ Z ” direction. By symmetry the magnitudes of the forces on “ ab ” and “ cd ” are the same but the directions are opposite and therefore cancel. The total magnetic force on “ $abcd$ ” is zero.



(c) The total magnetic field on “ ab ” is obtained by integrating the result found in part (b) from $x = R_1$ to $x = R_2$.

Physics 2402 Test #3 (11:30 Class) Fall 2004

1 (a) Uncharged gas molecules and charged carbon particles enter the exhaust pipe of a diesel engine at "A". The carbon particles have a mass of 3.00×10^{-10} (kg) and a positive charge of 2.50×10^{-10} (C). The gas molecules travel directly to "B" where they are discharged into the atmosphere. At "D" the carbon particles enter a region where there is a uniform magnetic field which is perpendicular to the page. The particles move in a quarter circle of radius 0.600 (m) (with center at "P") from "D" to "E" in the uniform magnetic field with a constant speed of 0.200 (m/s). Find the magnetic field in unit vector form assuming gravitational forces are negligible.



(b) Between "E" and "C" there is a uniform magnetic field having a magnitude of 0.200 (T) directed into the page. There is also a uniform electric field acting parallel to the Y axis. If the carbon particles move with the same speed of 0.200 (m/s) in a straight line from "E" to "C", find the electric field in unit vector form.

Solution:

1 (a) For the positively charged carbon particle to move from "D" to "E" along a path which is a quarter of a circle the magnetic force must act directly towards the center of the circle as shown in Fig. a. The magnetic force is:

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

The field \mathbf{B} is perpendicular to the page. By applying the RHR for the vector cross product we see that the correct direction for \mathbf{B} is out of the page to give the required direction for the magnetic force.

We apply Newton's II Law to the circular motion of the carbon particle:

$$\sum F_{rad} = m \frac{v^2}{R} \Rightarrow qvB \sin 90 = \frac{mv^2}{R}$$

$$B = \frac{mv}{qR} = \frac{3 \times 10^{-10} (.2)}{2.5 \times 10^{-10} (.6)} = 0.400(T)$$

The magnetic field is: $\vec{B} = .400\hat{k}(T)$

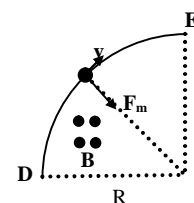


Fig. a

(b) Fig. b shows the carbon particle as it moves along a straight line from "E" to "C". The magnetic field is into the page and using the RHR we can determine that the magnetic force acts in the positive Y direction. The magnetic force is:

$$\vec{F}_m = q\vec{v} \times \vec{B} = qvB \sin 90 \hat{j}$$

The electric force acting on the carbon particle is: $\mathbf{F} = q\mathbf{E}$. For the particle to move in a straight line with a constant velocity the net force acting on it must be zero:

$$\vec{F}_m + \vec{F}_e = 0 \Rightarrow qvB\hat{j} + q\vec{E} = 0 \Rightarrow \vec{E} = -vB\hat{j} = -.040\hat{j}(N/C)$$

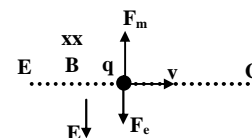


Fig. b

2 In the figure, "ABCD" is a wire carrying a current "I" in the direction shown by the arrows. The sections "AB" and "CD" are very long straight wires. The section "BC" is part of a circle of radius "R" with its center at the origin. The angle between the wire "CD" and the "X" axis is 45.0° . The direction of the positive "Z" axis is out of the page.

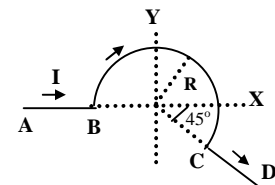
(a) **Derive**, using the Biot-Savart Law, the magnetic field (in unit vector form) at the origin produced by the current in the section "AB" of the wire.

(b) **Derive**, using the Biot-Savart Law, the magnetic field (in unit vector form) at the origin produced by the current in the wire "BC".

(c) Find the total magnetic field at the origin (in unit vector form) produced by the complete wire "ABCD".

Solution:

2 The magnetic field is obtained from the Biot-Savart Law:



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

The directions of the vectors $d\vec{s}$ and \vec{r} are shown in Fig. a. Since these vectors are parallel their vector cross product is zero and the magnetic field produced by this wire is zero.

(b) The directions of the vectors $d\vec{s}$ and \vec{r} are shown in Fig. b. The direction of the cross product of these vectors is found using the RHR and is into the page in the negative Z direction. The magnitude of $d\vec{s}$ is: $d\vec{s} = R d\theta$. The magnetic field at the origin produced by the current in "BC" is:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi/4} \frac{-\hat{k} R d\theta}{R^2} = -\frac{\mu_0 I \hat{k}}{4\pi R} \int_0^{\pi/4} d\theta$$

$$\vec{B} = -\frac{\mu_0 I \hat{k}}{4\pi R} [\theta]_0^{\pi/4} = -\frac{5\mu_0 I \hat{k}}{16R}$$

The unit vector " \hat{k} " and the radius " R " are constants and can be removed from the integral. The limits for the integration are zero to $(\pi + \pi/4)$.

(c) The field produced by the wire "CD" is zero since the angle between " $d\vec{s}$ " and the unit vector " \hat{r} " is 180° and therefore in the Biot-Savart Law the vector cross product " $d\vec{s} \times \hat{r}$ " is zero. The total field at the origin is:

$$\vec{B} = -\frac{5\mu_0 I \hat{k}}{16R}$$

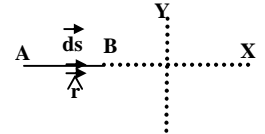


Fig. a

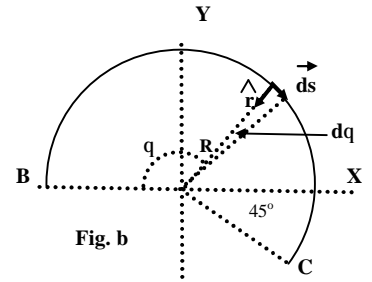


Fig. b

3 (a) A long straight cylindrical wire of radius " a " has a current " I " distributed uniformly over its cross section. **Derive**, using **Ampere's Law**, the magnitude of the magnetic field at a point inside the wire at a perpendicular distance " b " from the axis of the wire but not too close to either end.

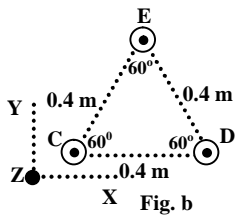


Fig. b

(b) Three long straight wires "C", "D" and "E" are shown in Fig. b. The wires are perpendicular to the plane of the page and have currents directed out of the page in the "+Z" direction. Wires "C" and "D" both have currents of 300 (A) while wire "E" has a current of 100 (A). The wires are located at the corners of an equilateral triangle of side 0.400 (m).

- Find the total magnetic field (in unit vector form) created by the currents in wires "C" and "D" at the location of wire "E".
- Find the magnetic force (in unit vector form) exerted on an 8.00 (m) length of wire "E" by the other wires.

Solution:

3 (a) We know from the symmetry that the magnetic field forms circles both inside and outside the wire. For the current direction shown, the direction of the magnetic field is shown. For Ampere's Law we choose the circular path "C" of radius " b " which is inside the wire. Along this path both " $d\vec{s}$ " and " \vec{B} " are parallel. Since the current is distributed uniformly across the cylinder the current density is:

$$J = \frac{I}{A} = \frac{I}{\pi a^2}$$

The current " I_{in} " that passes inside the path "C" is:

$$I_{in} = J(\pi b^2) = \frac{I b^2}{a^2}$$

We apply Ampere's Law over the path "C":

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \Rightarrow \int_C B \cos(0) ds = \frac{\mu_0 I b^2}{a^2}$$

$$B \int_C ds = \frac{\mu_0 I b^2}{a^2} \Rightarrow B(2\pi b) = \frac{\mu_0 I b^2}{a^2} \Rightarrow B = \frac{\mu_0 I b}{2\pi a^2}$$

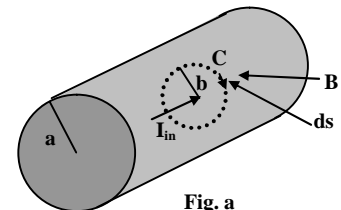


Fig. a

(b) The magnetic field of long straight wire forms circles about the wire. The convenient RHR gives the direction. The directions of the fields caused by the wires "C" and "D" at the location of "E" are shown. The magnitudes of these fields are the same:

$$B_C = B_D = \frac{\mu_0 I}{2pr} = \frac{4\pi \times 10^{-7} (300)}{2\pi (.4)} = 1.50 \times 10^{-4} (T)$$

The angle CEF is 60° and the angle between \mathbf{B}_C and CE is 90° . Also the angle between \mathbf{B}_D and DE is 90° . It follows that the angle between each magnetic field and the negative X axis is 30° as shown.

The "X" and "Y" components of the total field are:

$$B_x = -B_C \cos 30 - B_D \cos 30 = -2.60 \times 10^{-4} (T)$$

$$B_y = 0$$

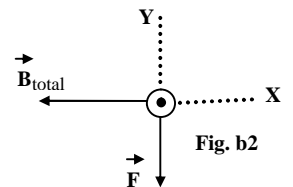
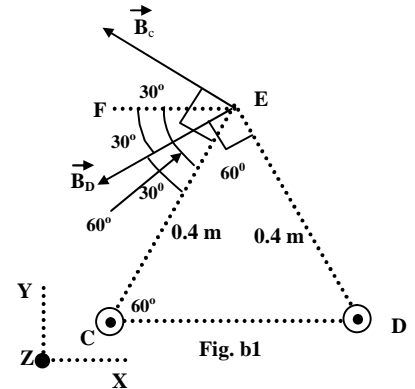
$$\vec{B}_{total} = -2.60 \times 10^{-4} \hat{i} (T)$$

The wire "E" is shown in Fig. b2. The vector "L" has a magnitude of 8.00 (m) and is directed out of the page. The direction of the force is found using the RHR. The force is in the negative "Y" direction.

The force on this length of wire has a magnitude:

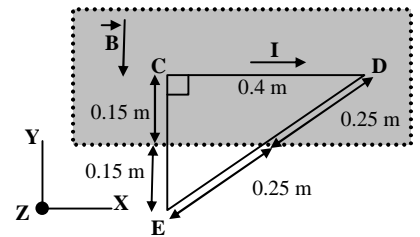
$$F = ILB \sin 90 = 100(8)(2.6 \times 10^{-4}) = 0.208 (N)$$

$$\vec{F} = -.208 \hat{j} (N)$$



Physics 2402 Test #3 (12:30 Class) Fall 2004

1 A right angled triangular loop of wire "CDE" has a current "I" (3.00 A) in the clockwise direction as shown. A uniform magnetic field "B" exists only in the shaded region. The magnitude of the field is 0.200 (T) and it is in the negative "Y" direction. The figure gives the lengths of wire that are inside and outside the magnetic field. The positive "Z" axis is out of the page.

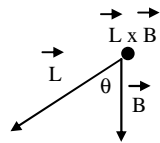


- (a) Find the magnetic force (in unit vector form) on the side "CD" of the loop.
 - (b) Find the magnetic force (in unit vector form) on the side "DE" of the loop.
 - (c) Find the magnetic force (in unit vector form) on the side "EC" of the loop.
- Solution:

1 The angle "θ" is found from the dimensions of the triangle: $\theta = \sin^{-1}(.4/.5) = 53.1^\circ$. We only need to find the forces on the lengths of wire that are inside the field region.

(a) The vector cross product required to find the direction of the force on "CD" is shown in Fig. a. Applying the RHR the direction of the force is into the page in the negative "Z" direction. The magnetic force is:

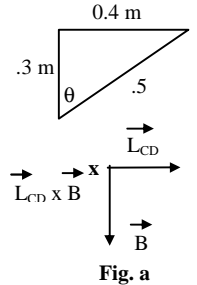
$$\vec{F}_{CD} = I\vec{L}_{CD} \times \vec{B} = IL_{CD}B \sin 90(-\hat{k}) = -3(.4)(.2)(1)\hat{k} = -.24\hat{k}(N)$$



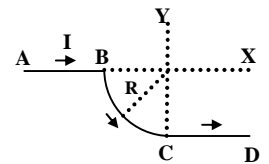
(b) The force on "DE" is just the force on the length of .25 m which is in the field. The vector cross product is shown in Fig. b and is in the positive "Z" direction, out of the page. The magnetic force is:

$$\vec{F}_{DE} = I\vec{L} \times \vec{B} = ILB \sin 53.1\hat{k} = 3(.25)(.2)(.8)\hat{k} = 0.120\hat{k}(N)$$

(c) On the side EC the angle between the length and the field vectors is 180° . The sine of this angle is zero so the magnetic force on this side is zero:



2 In the figure "ABCD" is a wire carrying a current "I" in the direction shown by the arrows. The sections "AB and CD" are very long and straight. The section "BC" is a quarter circle of radius "R" with its center at the origin. The direction of the positive "Z" axis is out of the page.



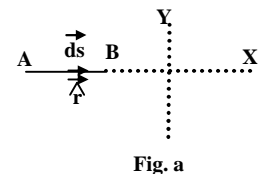
- (a) **Derive**, using the Biot-Savart Law, the magnetic field (in unit vector form) at the origin produced by the current in the section "AB" of the wire.
- (b) **Derive**, using the Biot-Savart Law, the magnetic field (in unit vector form) at the origin produced by the current in the quarter circle.
- (c) Find the field at the origin (in unit vector form) produced by the current in the wire "CD". You can use any formula on the equation sheet (*you do not have to derive your result from the Biot Savart or Ampere Laws.*)

Solution:

2 (a) The magnetic field is obtained from the Biot-Savart Law:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

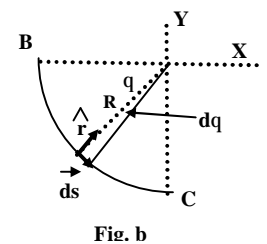
The directions of the vectors $d\vec{s}$ and \vec{r} are shown in Fig. a. Since these vectors are parallel their vector cross product is zero and the magnetic field produced by this wire is zero.



(b) The directions of the vectors $d\vec{s}$ and \vec{r} are shown in Fig. b. The direction of the cross product of these vectors is found using the RHR and is out of the page in the positive Z direction. The magnitude of ds is: $ds = R d\theta$. The magnetic field at the origin produced by the current in "BC" is:

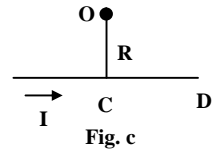
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{\hat{k} R dq}{R^2} = \frac{\mu_0 I \hat{k}}{4\pi R} \int_0^{\pi/2} dq$$

$$\vec{B} = \frac{\mu_0 I \hat{k}}{4\pi R} [q]_0^{\pi/2} = \frac{\mu_0 I \hat{k}}{8R}$$



The unit vector “ \hat{k} ” and the radius “ R ” are constants and can be removed from the integral. The limits for the integration are zero to $\pi/2$ since the wire is a quarter of a circle.

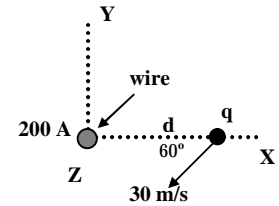
(c) Fig. c shows a very long wire whose midpoint is at “C”. The field of this wire can be found most easily from Ampere’s Law. The magnetic field is circular about the wire. Our convenient RHR is used to find the direction of the field by wrapping our right hand around the wire with the thumb extended in the current direction. At the origin, the field is out of the page in the positive “Z” direction. The field at the origin due to the section “CD” (which is one half the length of the wire shown in the diagram) must be half as large as the field of the wire shown. The field of the wire “CD at the origin is:



$$\vec{B} = \frac{1}{2} \left(\frac{\mu_0 I \hat{k}}{2pR} \right) = \frac{\mu_0 I \hat{k}}{4pR}$$

3 (a) A long, straight conducting cylinder of radius “a” has a uniform current density “J”. Use Ampere’s Law to derive the magnetic field at a point outside the cylinder a perpendicular distance “d” from its axis but not too close to either end.

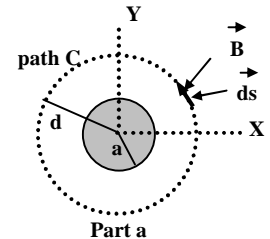
(b) In the diagram the “+Z” direction is out of the page. A very long straight wire is located along the Z axis (from minus to plus infinity) and is carrying a current “I” (200 A) in the positive “Z” direction. A positive charge “q” (6.00×10^{-4} C) is on the “X” axis a distance “d” (0.050 m) from the origin moving at that instant in the “XY” plane with a velocity “v” having a magnitude “v” (30.0 m/s). **This velocity vector is in the XY plane.** The angle between the velocity vector and the negative “X” direction is 60.0° . Find, at this instant, the magnetic force (in unit vector form) exerted on the charge by the current in the wire. A numerical value for the magnitude of the force is required in this part of the problem.



Solution:

3 (a) The magnetic field forms circles about the long wire which is shown looking into the wire from one end in the figure Part a. The convenient right hand rule is used to find the direction of the field. We curl our fingers around the wire with our thumb extended in the direction of the current. The field direction is shown in the figure.

We apply Ampere’s Law using the circular path “C” of radius “d”. Along this path the vectors $d\vec{s}$ and \vec{B} are parallel. The current inside this path is “ I_{in} ”. The current inside this path is the current in the cylinder which is found using the uniform current density: $I_{in} = J\pi a^2$.



Ampere’s Law gives:

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \Rightarrow \int_C B \cos 0 ds = \mu_0 I_{in}$$

$$B \int_C ds = \mu_0 I_{in} \Rightarrow B(2\pi d) = \mu_0 J\pi a^2 \Rightarrow B = \frac{\mu_0 J a^2}{2d}$$

In the above equations, B is a constant along the path and has been removed from the integral. The integral of “ds” over the circle is the circumference ($2\pi d$) of the circle.

(b) The figure shows an end view of the wire with the Z axis coming out of the page. By applying the convenient RHR by extending our thumb in the direction of the current and wrapping our fingers around the wire we see that the magnetic field at the location of the charge is in the positive “Y” direction.

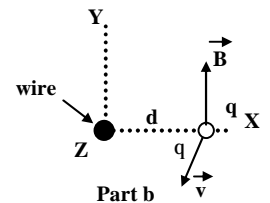
From Ampere’s Law the magnitude of this magnetic field is:

$$B = \frac{\mu_0 I}{2\pi d}$$

The magnitude of the force acting on the charge caused by this field is:

$$F = qvB \sin(\mathbf{q} + 90) = \frac{\mu_0 I q v \sin(\mathbf{q} + 90)}{2\pi R}$$

$$F = 7.20 \times 10^{-6} (N)$$



The direction of this force is obtained by determining the direction of the vector cross product of the velocity and the magnetic field. The RHR is used to find the direction of the vector cross product. Applying this rule we see that the direction is into the page in the negative "Z" direction.

$$\vec{F} = -7.20 \times 10^{-6} \hat{k} (N)$$

This force can also be obtained by writing the velocity and magnetic field vectors using unit vectors.

$$\vec{B} = B\hat{j}$$

$$\vec{v} = -v\cos(\mathbf{q} + 90)\hat{i} - v\sin(\mathbf{q} + 90)\hat{j}$$

$$\vec{F} = q\vec{v} \times \vec{B} = qvB(-\cos(\mathbf{q} + 90)\hat{i} - \sin(\mathbf{q} + 90)\hat{j}) \times B\hat{j} = qvB\sin(\mathbf{q} + 90)(-\hat{k})$$

$$\vec{F} = -7.20 \times 10^{-6} \hat{k} (N)$$