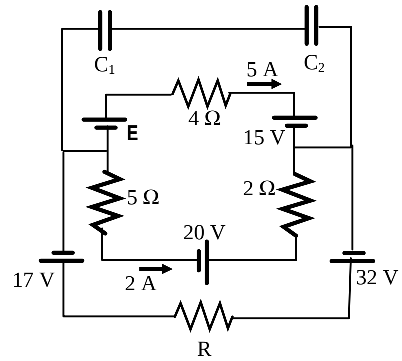
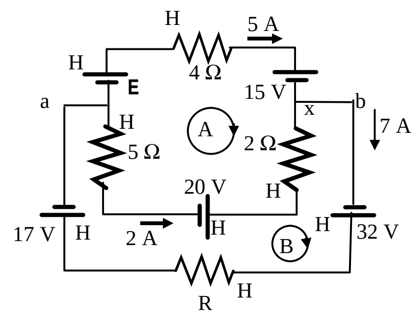
Physics 2402 Test #4 (Final Exam) Fall 2002

1 The circuit diagram shows several resistances and battery **Emfs**. Two currents are given along with their directions. The capacitors C_1 and C_2 are both fully charged.
(a) Find the current in the resistor labeled "R".
(b) Find "**E**".
(c) Find "R".
(d) Find the potential difference across the capacitor C_1 if the capacitances of C_1 and C_2 are 1.00×10^{-6} and 2.00×10^{-6} (F) respectively.

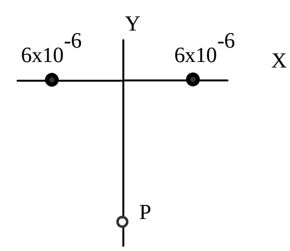


Solution:
(a) The capacitors are fully charged so no current exists in that part of the circuit and therefore are not shown in the figure. We apply Kirchoff's Rule 1 at junction "x". We let "I" be the current passing through the 32 V battery:
$\Sigma I_{in} = \Sigma I_{out}$ ⇨ $2 + 5 = I$ ⇨ $I = 7.00$ (A).
The direction of "I" is shown in the diagram.
(b) The high potential side of each battery and resistor has been marked. We apply Kirchoff's Rule #2 to loop A going in the CW direction:
$+\mathbf{E} - 5(4) - 15 + 2(2) - 20 + 2(5) = 0$ ⇨ $\mathbf{E} = 41.0$ (V)
(c) Apply Kirchoff's Rule #2 to loop in CW direction:
$+20 - 2(2) + 32 - 7R - 17 - 2(5) = 0$ ⇨ $R = 3.00$ (Ω)



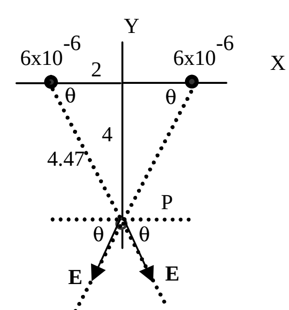
(d) The potential difference applied across the series capacitors is $V_b - V_a = -2(5) + 20 - 2(2) = 6.00$ (V)
Since C_1 and C_2 are in series, they can be replaced by the equivalent capacitor C':
$1/C' = 1/C_1 + 1/C_2$ and $C' = 0.667\times10^{-6}$ (F). The charge on C' is:
$Q' = C'\Delta V = 0.667\times10^{-6}(6) = 4.00\times10^{-6}$ (C). This is also the same as the charges on the series capacitors.
$\Delta V_1 = Q_1/C_1 = 4\times10^{-6}/1\times10^{-6} = 4.00$ V.

2 Two identical positive charges (6.00×10^{-6} C) are located at (-2,0) and (+2,0). These charges cannot move. Point "P" is at (0,-4) with all distances in meters.
(a) Find the total electric field at "P" in unit vector form.
(b) A negative charge (-3.00×10^{-6} C) is held at rest at "P". Find its initial acceleration when it is released. The mass of the charge is 8.00×10^{-4} (kg).
(c) Find the speed of the negative charge after it has moved a distance of 4.00 (m) from its initial position at "P".



Solution:
(a) From the given information we find the distance from either charge to "P" is 4.47 (m) and the angle "θ" is 63.4°.
The magnitude of the field of either charge at "P" is:

$$E = \frac{k|q|}{r^2} = \frac{9\times10^9\left(6\times10^{-6}\right)}{4.47^2} = 2.70\times10^3\left(\frac{N}{C}\right)$$



The X and Y components of the total field are:
$E_x = E\cos\theta - E\cos\theta = 0$
$E_y = -E\sin\theta - E\sin\theta = -2(2.7\times10^3)(\sin 63.4) = -4.83\times10^3$ (N/c)
The total field is: $\mathbf{E} = -4.83\times10^3\,\mathbf{j}$ (N/C)
(b) The electric force acting on a negative charge held at rest at "P" is:

F = q**E** = $(-3\times10^{-6})(-4.83\times10^{3}\mathbf{j}) = 1.45\times10^{-2}\mathbf{j}$ (N)
The acceleration of the charge is:
a = F/m = $(1.45\times10^{-2}\mathbf{j})/8\times10^{-4} = 18.1\mathbf{j}$ (m/s^2)
(c) Since the force is in the +Y direction, the charge will be at the origin after it has moved a distance of 4.00 (m) from its initial position. We let the initial position "i" be at "P" and the final position "f" be at the origin. The initial and final potentials due to both fixed charges are:

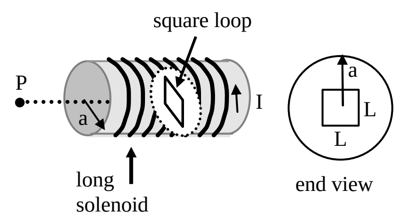
$$V_i = 2\left(\frac{kq}{r}\right) = 2\left(\frac{9\times10^9\left(6\times10^{-6}\right)}{4.47}\right) = 2.42\times10^4\,(V)$$

$$V_f = 2\left(\frac{kq}{r}\right) = 2\left(\frac{9\times10^9\left(6\times10^{-6}\right)}{2}\right) = 5.40\times10^4\,(V)$$

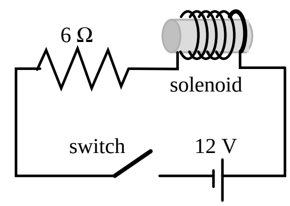
We apply the Energy Conservation principle to the motion of the ***negative*** charge from "P" to the origin:

$$\Delta K + q\Delta V = 0 \quad \Rightarrow \quad \frac{1}{2}mv_f^2 + q\left(V_f - V_i\right) = 0 \quad \Rightarrow \quad v_f = \sqrt{\frac{-2q\left(V_f - V_i\right)}{m}} = 14.9\left(\frac{m}{s}\right)$$

3 (a) A very long solenoid has a radius "a" and "n" turns per unit length. The magnitude of the magnetic field inside the solenoid is: $B = \mu_0 nI$. The current in the solenoid at any time "t" is: $I(t) = bt^2 + ct^4$. In this formula, "b" and "c" are positive constants. The direction of the current is shown. A single square loop of wire of side "L" and total resistance "R" is inside the solenoid as shown. The plane of the square loop is perpendicular to the axis of the solenoid. An end view of the solenoid and square loop is also shown. At time "t", find the magnitude of the induced current in the square loop.



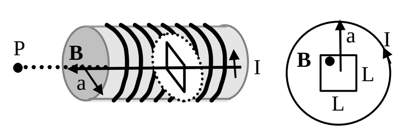
(b) As seen from point "P" state the direction of the induced current (CW or CCW). Explain your answer briefly.
(c) This part of the problem is not related to parts (a) and (b). A solenoid having negligible resistance, a 6.00 ohm resistor, a 12.0 (V) battery and a switch are connected as shown. The switch in initially open and it is then closed at the time t = 0. Exactly 7.00×10^{-4} (s) after the switch was closed the current passing through the resistor is 1.50 (A). Find the self inductance of the solenoid.



Solution:
(a) At time "t" the magnitude of the magnetic field inside the solenoid is:
$B(t) = \mu_0 nI(t) = \mu_0 n(bt^2 + ct^4)$.
The flux through the square loop is: $\Phi(t) = B(t)L^2\cos 0 = \mu_0 nL^2(bt^2 + ct^4)$.
The magnitude of the induced **Emf** at time "t" is:

$$\mathbf{E} = \left|-\frac{d\Phi}{dt}\right| = \left|-m_0 nL^2\,\frac{d\left(bt^2 + ct^4\right)}{dt}\right| = m_0 nL^2\left(2bt + 4ct^3\right)$$

The induced current is: $I(t) = \mathbf{E}/R = 2\mu_0 nL^2R^{-1}(bt + 2ct^3)$
(b) We apply Lenz's Law to find the direction of the induced current. In the end view figure (as seen from "P") the flux out of the page is increasing since the applied current and therefore the magnetic field are increasing with time. The induced magnetic field must be into the page to oppose the change in flux which is occurring. Using the RHR of convenience we find the induced current in the square loop must be CW as seen from "P" to produce an induced field into the page.
(c) The current in the LR circuit at a time "t" after switch is closed is:
$I(t) = (\mathbf{E}/R)(1 - e^{-t/\tau})$
Using the given information we have:

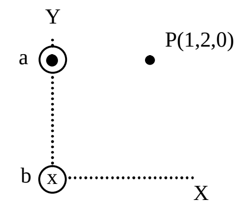


$$1.5 = 2\left(1 - e^{-\frac{t}{t}}\right) \quad \Rightarrow \quad e^{-\frac{t}{t}} = .25 \quad \Rightarrow \quad -\frac{t}{t} = \ln(.25)$$

Using the given time we obtain for the time constant: $\tau = 5.05\text{x}10^{-4}$ (s).
The inductance is obtained from: $\tau = L/R$ or $L = \tau R = 3.03\text{x}10^{-3}$ (H).

4 (a) A very long straight cylindrical conductor of radius “a” has a uniform current “I”. <u>**Derive**</u>, using Ampere’s Law, the magnitude of the magnetic field <u>**inside**</u> the cylinder at a point which is a perpendicular distance “r” from the axis (r < a).

(b) This part of the problem is not related to Part a. Two long conducting wires shown as “a” and “b” are perpendicular to the page. Wire “a” has a current out of the page while wire “b” has a current into the page as shown. Wire “b” passes through the origin while wire “a” passes through (0,2,0) where all distances are in meters and the +Z axis is out of the page.



(i) The Y and Z components of the total magnetic field at P(1,2,0) caused by the current in both wires are zero. The current in wire “a” is 30.0 (A). Find the current in wire “b”.
(ii) Find the total magnetic field at “P” in unit vector form.
(iii) A small, positively charged sphere is moving with a constant velocity of $5.00\text{x}10^{4}$ **k** (m/s) along a line which is parallel to the Z direction and passes through point “P”. The charge on the sphere is $4.00\text{x}10^{-4}$ (C) and its mass is unknown. The earth exerts a gravitational force on the charge in the negative Y direction. Find the mass of the sphere.

Solution:

(a) IN Fig. a it has been assumed that the current direction is out of the page. By symmetry we know that the field forms circles about the axis of the cylinder. Using the RHR of convenience we find that the direction of the field which is shown in the figure. We choose a circle of radius “r” to evaluate the line integral in Ampere’s Law. Along this path **ds** and **B** are parallel. The current density is: $J = I/A = I/(\pi a^2)$. Amperes Law gives:

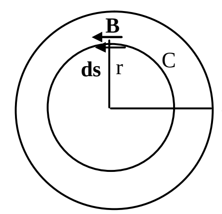


Fig. a

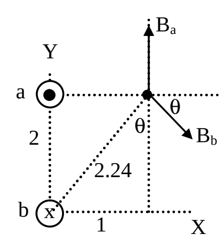
$$\int_C B\cos 0\, ds = m_0\left(I_{insideC}\right) \quad \Rightarrow \quad B\int_C ds = m_0\left(Jpr^2\right) \quad \Rightarrow \quad B(2pr) = m_0\left(\frac{I}{pa^2}pr^2\right)$$

The magnitude of the magnetic field is: $$B = \frac{m_0 Ir}{2pa^2}$$

(b) The distance from wire “b” to “P” is 2.24 (m) and the angle “θ” is 26.6°. The magnitudes of the magnetic fields at “P” are:

$$B_a = \frac{m_0 I}{2pr} = \frac{4p \times 10^{-7}(30)}{2p(1)} = 6.00\times10^{-6}(T)$$

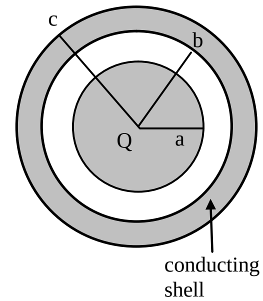
$$B_b = \frac{4p \times 10^{-7}(I_b)}{2p(2.24)} = 8.93\times10^{-8} I_b$$



(i) The Y component of the total field is zero:
$B_a - B_b \sin 26.6 = 0$ ⇨ $6\text{x}10^{-6} - 8.93\text{x}10^{-8}\, I_b(\sin 26.6) = 0$
The current in wire “B” is: $I_b = 150$ (A).
(ii) The X component of the total field is:
$B_x = 0 + B_b\cos 26.6 = 8.93\text{x}10^{-8}(150)\cos 26.6 = 1.20\text{x}10^{-5}$ (T)
The total field is: **B** = $1.20\text{x}10^{-5}$ **i** (T)
(iii) The magnetic force on the charge is:
$\mathbf{F}_m$ = q**v x B** = $4\text{x}10^{-4}$ ($5\text{x}10^{4}$**k**) **x** ($1.2\text{x}10^{-5}$ **i**) = $2.40\text{x}10^{-4}$ **j** (N)
The gravitational force on the charge is: $\mathbf{F}_g$ = -mg **j** (N)
Since the charge moves with a constant velocity the acceleration is zero. Apllying Newton’s II Law gives:

$2.4 \times 10^{-4} - 9.8m = 0 \Rightarrow m = 2.45 \times 10^{-5}$ (kg)

5 A spherical charge distribution has a positive charge “Q” distributed uniformly throughout the volume of a sphere of radius “a” as shown. This spherical charge distribution is surrounded by a concentric, conducting, spherical shell of inner radius “b” and outer radius “c”. The conducting shell has a total negative charge of “-3Q”.
(a) **Derive** the electric field inside the spherical charge distribution at a distance “r” from its center (r < a).
(b) Find the total charge on the outer surface of the conducting shell (at r = c).
(c) **Derive** the magnitude of the electric field at a point outside the conducting shell a distance “r” from the center of the spherical charge distribution (r > c). Write this field in unit vector form using the unit vector **r** which points radially outwards from the center of the spherical charge distribution.



Solution:

5 (a) The volume charge density inside the sphere of radius “a” is:

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{3Q}{4\pi a^3}$$

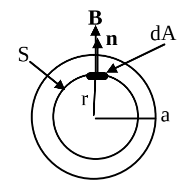


Fig. a

We choose a Gaussian sphere of radius “r” (r<a). On this surface **B** and **n** are parallel since, by symmetry, the field is radial. The charge inside the surface “S” is ρV where “V” is the volume of the sphere of radius “r”. Gauss’ Law gives:

$$\int_S E\cos 0\, dA = \frac{1}{\varepsilon_0}\left(\left(\frac{3Q}{4\pi a^3}\right)\left(\frac{4\pi r^3}{3}\right)\right) \Rightarrow E(4\pi r^2) = \frac{Qr^3}{\varepsilon_0 a^3} \Rightarrow E = \frac{Qr}{4\pi\varepsilon_0 a^3}$$

(b) We choose a Gaussian surface “S” which is inside the conductor where the field is zero. The charge on the inside surface of the conductor (radius “b”) is Q_b. Applying Gauss’ Law over this surface gives:

$$\int_S 0\, dA = \frac{1}{\varepsilon_0}(Q + Q_b) \Rightarrow Q_b = -Q$$

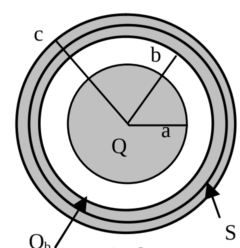


Fig. b

Since the total charge on the conductor is -3Q we have:
$Q_b + Q_c = -3Q \Rightarrow Q_c = -3Q - (-Q) = -2Q$

(c) The field for r > c is found using a Gaussian sphere of radius “r” as shown in Fig. c. The charge inside this surface is +Q – 3Q or -2Q. Since the net charge is negative the electric field is radially inwards and the angle between **E** and **n** is 180°.

$$\int_S E\cos 180\, dA = \frac{1}{\varepsilon_0}(-2Q) \Rightarrow -E(4\pi r^2) = -\frac{2Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 r^2}$$

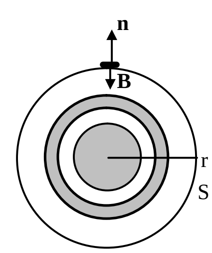
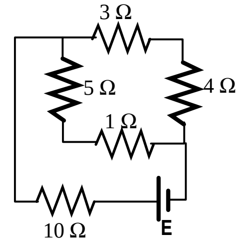


Fig. c

The field is:

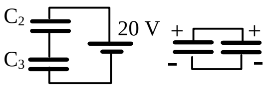
$$\vec{E} = -\hat{r}\left(\frac{Q}{2\pi\varepsilon_0 r^2}\right)$$

6 (a) (i) Find the equivalent resistance of the circuit shown.



Part a

(ii) The 1, 3, 4 and 5 ohm resistors can each dissipate as heat a maximum power of 20 watts without being destroyed. The 10 ohm resistor is very large and it is assumed that it can dissipate any amount of heat without being destroyed. What is the maximum possible value of the battery Emf, **E**, that can be used without destroying any of the resistors?

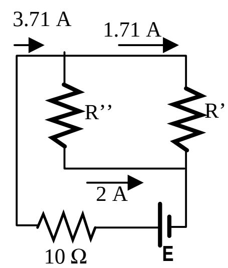


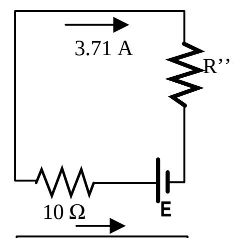
Part b

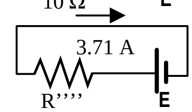
(b) The capacitors C_2 (2.00×10^{-6} F) and C_3 (3.00×10^{-6} F) are connected to a 20.0 volt battery as shown. The capacitors are disconnected from the battery and from each other without disturbing the charges on either capacitor. The capacitors are then connected as shown with the plates having like charges connected together. Find the final charge on C_3.

Solution:

(a) (i) The 3 and 4 ohm resistors are in series and have been replaced by R’ while the 5 and 1 ohm resistors are in series and have been replaced by R’’:







R’ = 3 + 4 = 7 (Ω) and R’’ = 1 + 5 = 6 (Ω)

The resistors R’ and R’’ are now in parallel and can be replaced by R’’’:

1/R’’’ = 1/R’ + 1/R’’ ⇨ R’’’ = 3.23 (Ω)

Finally, R’’’ and the 10 ohm resistor are in series and can be replaced by:

R’’’’ = R’’’ + 10 = 13.2 (Ω).

(ii) Since R’ and R’’ are in parallel they have the same potential difference. R’’ will have a larger current since R’’ < R’. The power produced in a resistor is I^2R so the 5 ohm resistor will produce the largest power since it has the largest current and resistance. The maximum safe current in the 5 ohm resistor (and R’’) is: $I_{R''} = (P/R)^{1/2} = (20/5)^{1/2} = 2.00$ (A).

The potential difference across R’’ and R’ is: $\Delta V = I_5 R'' = 2(6) = 12$ (V).

The current through the 3 and 4 ohm resistors (and R’) is: $I_{R'} = \Delta V/R' = 12/7 = 1.71$ (A)

By applying Kirchoff’s Rule #1 at the junction we find that the current in the battery part of the circuit is:

I = 1.71 + 2.00 = 3.71 (A).

This current is shown in the figures. From the last figure we have from Ohm’s Law (or Kirchoff’s Tule #2):

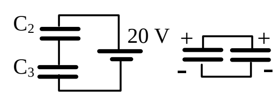
E = IR’’’’= 3.71(13.2) = 49.0 (V).

(b) Since the capacitors are in series they can be replaced by an equivalent capacitor C’:

$1/C' = 1/C_2 + 1/C_2$ a $C' = 1.20 \times 10^{-6}$ (F). The charge on this capacitor (and C_2 and C_3) is:

$Q' = Q_2 = Q_3 = C'\Delta V = (1.2 \times 10^{-6})(20) = 2.40 \times 10^{-5}$ (C).

When the capacitors are connected as shown in the second figure, the potential difference across each is the same and the new charges are Q'_2 and Q'_3.



Part b

$\Delta V_2 = \Delta V_3$ ⇨ $Q'_2/C_2 = Q'_3/C_3$ ⇨ $Q'_2 = 0.667 Q'_3$.

Since the total charge did not change:

$Q_2 + Q_3 = .667 Q'_3 + Q'_3$ ⇨ $Q_3' = 2(2.4 \times 10^{-5})/1.667 = 2.88 \times 10^{-5}$ (C).